BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 3, Number 1, July 1980

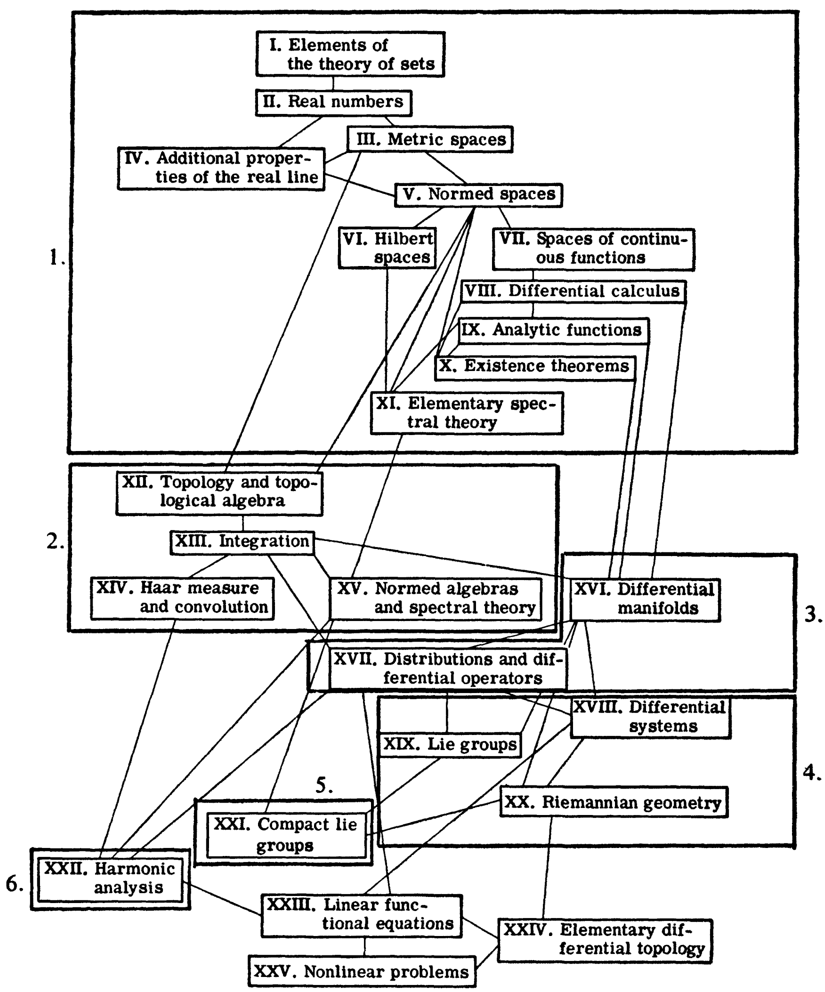
0002-9904/80/0000-0305/$02.25

BOOK REVIEWS

Treatise on analysis, by Jean Dieudonné, translated by I. G. MacDonald, Academic Press, New York, vol. I, 1969, xviii + 387 pp., $13.95; vol. II, 1970, xiv + 422 pp., $24.00; vol. III, 1972, xvi + 388 pp., $25.00; vol. IV, 1974, xv + 444 pp., $34.00; vol. V, 1977, xii + 243 pp., $21.50; vol. VI, 1978, x + 239 pp., $25.00.

1. The scope. Dieudonné is putting together a monumental treatise which will comprise d volumes; the best estimate we know on d is $8 < d < 12$, but the upper limit is not certain. So far, 6 volumes have appeared in English and 2 others in French that have not yet been translated.

SCHEMATIC PLAN OF THE WORK



The main topics treated, along with the relevant volumes are reproduced below in the author's schematic plan of the work. This plan illustrates a basic theorem in mathematical writing which even experts find hard to swallow: *to find the true length (measured in pages or years) of a writing project, multiply your initial estimate by a factor of at least three*. In fact, if one takes on a project with large enough scope, the writing process can become stationary: *even though you write feverishly, your project is at all times only half-completed.* Dieudonné's position is probably almost as bad as the theorem indicates (he originally estimated 4 volumes) and could even be as bad as stationary, depending on what is meant by "nonlinear problems". If he means to include topics like degree theory, calculus of variations, bifurcation theory and nonlinear partial differential equations, which are accepted topics in nonlinear analysis, then stationarity is a definite possibility.

Dieudonné uses the word "analysis" in a definite sense; he largely means those parts of analysis that interact strongly with differential geometry and algebra. Thus, he includes thorough discussions of differential calculus in Banach spaces, Lie groups, differentiable manifolds, measure theory, C^* algebras, spectral theory and harmonic analysis, but does not discuss in any depth topics like unbounded operators that generate semigroups, convex analysis, manifolds of mappings "classical" analysis (elliptic estimates, Hilbert transforms, weak convergence in L^p spaces, etc.), variational inequalities, etc.

2. The audience. One of the central messages in Halmos' *How to write mathematics* (American Mathematical Society publication) is *write for a clearly defined audience*. This issue is especially important for a work of this scope. Ideally, the books are written for aspiring graduate students or mature mathematicians eager to learn the field. They are not designed to be completely thorough with the latest results but rather give carefully selected topics. Just how realistic is this goal? To answer this question we begin by putting the work in over-all perspective.

There seem to be three broad types of books that contribute to progress in mathematics:

1. Books of synthesis and exposition;
2. Reference works;
3. Research monographs.

Books in category 1 tend to be of large scope and give the authors' own perspective on a field. They often simplify and synthesize existing knowledge into more digestible form and allow mathematicians to keep pace with expansions in the field; these mathematicians are often younger, so this category is crucial to mathematical evolution. A good example is *Gibbs' lectures on vector analysis* by E. B. Wilson in 1900; this book first made available the basic results of vector analysis without the necessity of learning quaternions first. Category 1 also includes (not necessarily finished) works that attempt to synthesize different areas, such as some of the books of R. Hermann [reviewed in Bull. Amer. Math. Soc. **79** (1973), 1151–1162].

Books in category 2 are polished works of a well developed and firmly established area of mathematics that are used regularly by researchers.

Examples are the books of Dunford and Schwartz *Linear operators* and Spanier's *Algebraic topology*. These books do not necessarily present the latest results, but do intend to be complete within their given domain.

Category 3 consists of books which are usually narrower in scope than those in category 2, but go deeper and discuss current research; they often go out of date quickly. An example is J. Schwartz' *Nonlinear functional analysis*.

Dieudonné's work is about 3/4 in category 1 and about 1/4 in category 2. The reason it is not more in category 2 is that he has "systematically tried not to be exhaustive". The book is primarily aimed at the middle ground between the "minimum tool kit" and a "specialized monograph". This leads us to suppose that the typical reader will be a graduate student aspiring to be a research mathematician.

How is such a reader to go about the task? With volume I it was simple . . . it makes a beautiful one semester course. Should this student now plow through all the volumes? The answer seems to be yes, for the simple reason that the volumes are so tightly integrated that there is little alternative.

For example, suppose you wish to read about Bochner's theorem on functions of positive type. You begin by consulting the index; this process takes several minutes, as the index lists only the section number. Several more minutes of chasing references like 22.10.10.6 (references with more than 2 digits should be banned!) leads you to realize that the proof is actually scattered through 3 volumes. This situation is not exceptional.

A few of the best graduate students could plow through all these volumes. A fellow graduate student (and now a professor) at Princeton purportedly did all the problems in Dunford and Schwartz!. However, we all differ in what we believe to be important; I would not ask one of my students to read *all* of these volumes, simply because for me the ground work has a slightly different emphasis . . . somewhat more "hard" analysis. This means that I only desire certain chapters and would use other references for other topics. The inflexibility of Dieudonné's treatise makes this difficult.

Here is an example of what could happen. Suppose a reader wishes to learn the *basic background* for a key result in analysis, let us say the Hodge theorem or the Atiyah-Singer index theorem. Would they go to Dieudonné? For parts of the background, certain chapters (like XVI, XVII, XX and XXIV) are very appropriate, but key bits are also missing (elliptic theory, characteristic classes). [It is conceivable of course that material of this sort will all go into volumes yet to appear.] At the moment, this type of reader is probably better off consulting a narrower research monograph and consulting a variety of background references (including Dieudonné) as needed.

For advanced graduate students and working mathematicians who already know parts of the material and wish to learn others, the situation is better. For example, it is not too hard to jump right into volumes V or VI. The main problem is picking up the notation, some of which is nonstandard. For example, pull-back is denoted ${}^t\!f$ rather than the standard notation f^*, he views the universal covering group of $SO(3)$ to be $U(1, H)$ rather than the more familiar $SU(2, \mathbf{C})$, etc.

In graduate level mathematics there are two main teaching philosophies which, to some extent, distinguish books in categories 1 and 2 from 3. The first is to provide broad competence with considerable depth in the proposed area of research. The second is to provide a shallower but broad base and then make a bee-line for the frontier of knowledge in the proposed subject. *In text-book writing these two goals are incompatible.* The examples given indicate that Dieudonné's books fit well with the first philosophy, but in some cases, not with the second.

We thus conclude that the appropriate audience for this book consists of: 1. ambitious graduate students who wish a broad base in analysis, who are willing to supplement the work with other books or expert guidance, and who do not necessarily desire to get to a research topic quickly and 2. Mature mathematicians who wish to fill in a gap in their background and who are willing to back reference to pick up notation on standard theorems they may know.

3. The topics. In my view, the strongest feature of these volumes is the balance of topics between geometry and analysis. This is a fruitful area and there are not many authors capable of bridging the gap. Other books with a similar interdisciplinary nature are Berger's *Nonlinearity and functional analysis*, Choquet-Bruhat, DeWitt and Morette's *Analysis, manifolds and physics*, Palais' *Seminar on the Atiyah-Singer Index Theorem* and Wells' *Analysis on real and complex manifolds.*

There are too many mathematicians entrenched in their own methods, protecting their domain with notational armor. Dieudonné is one of those capable enough to break through these primitive instincts. This is why they are great category 1 books.

The specific choice of topics has been greatly influenced by Bourbaki and the author's own tastes rather than systematic needs of mathematicians. This is, of course, understandable and excusable. Dieudonné makes no secret of his choices; they are laid out in *Present trends in pure mathematics*, Advances in Math. **27** (1978), 235–255 and *Panorama des Mathématiques Pures. Le choix Bourbachique*, Dunod (1977) [Reviewed in Bull. Amer. Math. Soc. (N. S.) **1** (1979), p. 678].

One factor which seems important in determining useful present trends is the interconnections between diverse fields within mathematics. Dieudonné brings out samples of this very nicely. However Dieudonné largely ignores the interaction between pure and applied mathematics. *History screams at pure mathematicians not to ignore applications*; the origins of such important topics as calculus, Fourier series, operator theory and dynamical systems were all closely related to applications. Some of the greatest mathematicians, Gauss, Hilbert, Poincaré, and von Neumann, were all rooted in applications. A modern example is the deep and beautiful applications of geometry to gauge theories (the paper of Atiyah, Hitchin and Singer, Proc. Roy. Soc. London **362** (1978), 425–461 is representative).

Good books have a coherent set of topics of even level and sophistication of presentation. Dieudonné is excellent in this respect with the following exceptions. In volume 1, differential calculus in Banach spaces is developed. In the past decade, many impressive applications of infinite dimensional manifolds have been given. It seems strange that Dieudonné then reverts to finite dimensions when discussing manifolds. Why the sudden jump? Some topics, such as flows and Lie derivatives are too far apart in the text. On the positive side, Dieudonné avoids excessive and useless generality in keeping with modern trends, and tries to keep the needed tools to a minimum. S. K. Berberian's review of volume II discusses this point (Math. Reviews **38** (1969), #4247); his summary is worth repeating:

> "Bourbaki gets to deep water only after constructing a fleet of nuclear submarines; the author has decided to get there by rowing, in a glass-bottomed boat. Planned and executed with consummate skill, this work should have an impact on mathematical pedagogy comparable in magnitude to the impact of Bourbaki on mathematics."

4. Orientation. Well written books properly orient their readers with good introductions, and good textual remarks on what is important, what is coming and what has been done. As far as chapter introductions are concerned, Dieudonné is superb. For instance, the introduction to harmonic analysis (volume VI) gives even the nonexpert an understanding of what the subject is about and a bit of history.

On textual aids Dieudonné is a bit uneven; sometimes the books have an excellent sense of direction, other times it is jumbled. Some negative examples are: Stokes' theorem is never stated cleanly, the Gauss-Codazzi equations and Frobenius theorem are buried and go unnamed, Nash's embedding theorem is not attributed to Nash, periodic geodesics are discussed but with no references for more information, Choquet's convexity proof of Bochner's theorem is not mentioned, no reference to the Newlander-Nirenberg theorem or Darboux's theorem is given in the chapter on differential systems, no discussion is given of why the Cauchy-Kovalevska theorem is inappropriate for most evolution problems (Hadamard's basic message), no mention is made in the dangling section on Sobolev spaces that even some of the most basic Sobolev inequalities are not covered, and de Rham's name does not appear in the discussion of currents. Dieudonné is often ruthless concerning backwards orientation; for example, the first paragraph of text in Volume VI is brutal... one had better have Chapters XXI and XV in tip-top shape before going on.

5. Exercises and examples. The exercises are largely good and well selected. Some seem a little too hard; for example, he asks the reader to prove the Denjoy-Carleman theorem in XXII §19 (volume 6) and to solve Hilbert's fifth problem in XIX §8 (volume 4). Is this an excuse to cram in more material or are they really useful, doable exercises? Some exercises are not properly referenced; for example, I believe Exercise 1 on p. 11, Volume IV is a theorem due to (amongst others) Brezis. Palais' theorem on Lie groups defined by invariance of geometric structures loses its point in an exercise.

Good books frequently come down to earth with *concrete, self-contained, simple* examples to illustrate the main points. Dieudonné is largely good, although occasionally uneven. Sometimes there are long sections with no illustrations; othertimes the examples are really excellent, such as the illustration of singular integral manifolds on p. 98 of Volume IV.

Authors' and lecturers' insensitivity to the needs of most readers and listeners for basic down-to-earth examples is one of the greatest sins mathematicians commit and only tends to reinforce barriers between disciplines. For example, it is now evident that the subjects of bifurcation theory and singularities of mappings have a great deal in common and a great deal to offer each other; a brick wall of noncommunication and reticence took almost a decade to break down. (Nirenberg's Courant Institute notes on Nonlinear Analysis were instrumental in this bridging.) There *is* room for improvement in Dieudonné's treatise in this respect.

6. Overview. Most mathematicians write advanced books for themselves; to set down their views for the record, to educate a close circle of followers or simply for their own ego, prestige or promotion. It is a rare mathematician who is earnest about making the necessary effort to break down barriers and to further mathematical evolution by teaching aspiring mathematicians with sensitivity and understanding. I believe Dieudonné is one of this rare breed.

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Infinite dimensional linear systems theory, by Ruth F. Curtain and Anthony J. Pritchard, Lecture Notes in Control and Information Sciences, vol. 8, Springer-Verlag, Berlin-Heidelberg-New York, 1978, viii + 297 pp., $14.80.

As I have been interested in this area for a number of years, it may be significant to say at the outset that this is a book I wish I had written myself. The book literature in infinite dimensional linear systems, or distributed parameter systems, to use the favored engineering terminology, is not very extensive. Perhaps the first significant contribution to that select category was A. G. Butkovsky's *Theory of optimal control of distributed parameter systems* (American Elsevier Publ. Co., New York, 1969; the Russian version appeared in 1965). This pioneering work was followed by J. L. Lions' landmark volume *Optimal control of systems governed by partial differential equations* (Springer-Verlag, New York, 1971; the French version appeared in 1968). There have, of course, been numerous journal articles, published conference proceedings, bibliography listings and review articles. In one of the last (SIAM Review, Vol. 20, 1978) I took some pains to point out that there is a distinct difference between the notions and methodology of *optimal* control and those of general control systems theory. The latter is a study of dynamical systems which involve control parameters explicitly intended for use in modifying systems behavior together, usually, with a set of admissible observation functionals providing information on the system state. Systems theory primarily involves such concepts as controllability, stabilizability, observability, etc. While the Butkovsky and Lions contributions do not entirely neglect these basic systems theory concerns, I think it is fair to say in both cases that the emphasis was placed on optimization. Thus it is particularly gratifying to be able to welcome the work by Curtain and Pritchard which, while not neglecting