methods. These arise for example in plate bending problems where energy has a form similar to (3). The standard approximation thus requires that admissible displacements have square integrable second derivatives, and this can cause practical problems for piecewise polynomial functions in many cases. In the nonconforming approach one, in essence, ignores this continuity constraint thereby obtaining a space U^h which is not a subset of U. This will of course not work in general, and the approach required a careful and systematic analysis. Ciarlet has done exactly this, and his treatment of this subject is unequaled.

The only disappointing chapter is the one on mixed methods. The latter are based on variational principles where solutions emerge as stationary points rather than minima as in (2). The author's error analysis uses a generalized Lax-Milgram approach. Invariably continuity requirements of the latter lead to unusual norms that obscure important structural properties of the error (e.g., optimality or suboptimality of the rate of convergence in L_2). This, however, should not be regarded as a major defect of this book since the chapter is short and since the author in the preface acknowledges that he did not wish to stress mixed methods.

The following quotation from P. R. Halmos precedes Chapter I. "A mathematician's nightmare is a sequence n_ϵ that tends to 0 as ϵ becomes infinite." Ciarlet has heeded the message here for his choice of notation is excellent and apparently carefully planned to aid the reader through the more technical material. There are a few misprints but they are minor and do not detract. Finally the notes concluding each chapter are balanced and very informative.

In short, this is an excellent, well written, and, for the most part, carefully planned book that deserves study by anyone working in the general area of finite element methods.

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Non-Archimedean functional analysis, by A. C. M. Van Rooij, Pure and Applied Math., vol. 51, Marcel Dekker, New York, 1978, ix + 404 pp., $29.50.

There are fields that are complete, locally compact, have a nontrivial valuation (absolute value) and they do not have to be the real or complex number field to be interesting to the analyst, be he harmonic, classical or functional. And this is so in spite of the fact that such a field that is not **R** or **C** will be totally disconnected and non-Archimedean. An example that is of characteristic zero is a p-adic number field or a finite algebraic extension of a p-adic field. An example that is of finite characteristic is a field of formal Laurent series over a finite field. This is, in fact, a complete list of such fields. They are usually referred to as local fields and they are non-Archimedean in the sense that the absolute value satisfies the strong triangle inequality: $|x + y| \leqslant \{|x|, |y|\}$.

While it might appear that functional analysis is the study of certain interesting kinds of topological vector spaces (for example: Banach spaces, Banach algebras or Fréchet spaces) it is truly about the interesting classes of functions and their generalizations (L^p spaces, spaces of continuous and continuously differentiable functions, measures, distributions, spaces of operators between these spaces) that give rise to the presumed primary objects of functional analysis. It is from a consideration of such "function spaces" that we discover what natural properties our vector spaces will enjoy. Thus since we have been taught that the only spaces that are really of interest are real or complex functions spaces it follows that we will believe that the only vector spaces of interest will be those over **R** or **C**. Furthermore, with few exceptions, we are expected to define our function spaces over domains that are, at least, locally Euclidean.

Of course, we all know that local fields, in particular p-adics, are of some interest. After all they are important in number theory and they provide all sorts of examples in algebra and representation theory. But, if you are going to do "real analysis" then do real analysis; that is, define your function class on a Euclidean manifold and let your "function class" be real or complex valued; and if you are a little more courageous you can let them be vector valued, or Banach spaces valued, but of course, if the vector space or Banach space is over the reals or the complex field. Now you can violate these strictures if you will; you will not lose your standing as a mathematician, but pretend you are just taking a modest fling, or be prepared for aggravation.

It is relatively safe to break these rules about where the functions are defined. Many of us have been doing harmonic analysis for complex valued functions defined on local fields and related non-Archimedean domains. At an absolute minimum, it seems to be reasonable to try to find out more about the classical situation by seeing what remains the same, what is gained and what is lost. Even so, one is at relative risk. Consider: "Say, when are you going to write up *everything there is to know* and get back to doing interesting mathematics?" While the emphasis is mine, the question, *not* rhetorical, is genuine.

Those studies, which I took to be somewhat courageous, pale in the light of the studies reported in the book under review. It is the most recent report from the "*P*-adics and Dining Society" (charter members: Marcus van der Put, A. C. M. van Rooij, Wim Schickof and Jan van Tiel). They are interested in the functional analysis that develops when the underlying function spaces are K-valued, K a local field. Their work establishes that there is a lot of interesting mathematics in this area.

This current contribution is a text book and is extra-ordinarily complete, in the areas with which it deals: An introduction to requisite valuation theory and topology; Banach spaces (spherical completeness, orthogonal bases); Banach algebras; Integration; Invariant ("Haar-like") measures; and a brief look at the Fourier theory. It is filled with exercises, outlines of unsolved and partially solved problems and excellent notes. The author makes no attempt to be encyclopedic; avoiding topics such as: special functions, categories of Banach spaces, analytic functions and rings of power series. He does, however, give some references to the literature for those topics.

The list of references is long and overly complete in some ways and is lacking in others. On the plus side we have that the notes and comments are well and specifically referenced. However, the introduction refers to some work that is not referenced and an occasional reference seems to be included because it has the right words in the title and not because it is used or refers to any subject discussed.

The fashionable view is that if functional analysis is not Archimedean then it is either a trivial extension of the Archimedean case which holds because of certain abstract nonsense, or it is trivial because it clearly fails or holds for the most elementary of reasons. Consider some counter-examples: If G is a nice enough non-Archimedean group and $\hat{G}$ is its dual (by the way, there are *three* natural notions of dual available) then the space of finite measures (non-Archimedean valued measures!) on G is isomorphic with space of bounded uniformly continuous functions on $\hat{G}$. For Banach algebras, the rather spectacular failure of the Gelfand-Mazur theorem leads to *four* non-equivalent notions of the spectral radius, and it would seem that the future holds, not a general theory of Banach algebras, but a variety of such theories.

If you have the patience to read a text book, and it takes patience to read a text, you will find that the view that non-Archimedean functional analysis is trivial is not entirely correct.

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Quasi-ideals in rings and semigroups, by Ottó Steinfeld, Akadémiai Kiadó (Publishing House of the Hungarian Academy of Sciences), Budapest, 1978, xii + 154 pp.

The notion of a quasi-ideal was first introduced by the author, for rings in 1953 and for semigroups in 1956. An additive subgroup Q of a ring A is called a *quasi-ideal of A* if $QA \cap AQ \subseteq Q$. The same definition applies if A is a semigroup, changing "additive subgroup" to "non-empty subset". More than fifty papers have appeared since that time, dealing with quasi-ideals, and the present book gives a systematic survey of the main concepts and results in this area. The author himself is the outstanding contributor, with more than a dozen papers on the subject.

A subset of a semigroup S is a quasi-ideal of S if and only if it is the