and Schwartz: Riesz operators, generalized hermitian operators, prespectral (as opposed to spectral) operators, and well-bounded operators. The exposition is well knit, and there are numerous examples. Dowson's book is a fine contribution to the literature, and should benefit both experts and novices. If there is any shortcoming, it would be the absence of exercises.

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Bessel polynomials, by Emil Grosswald, Lecture Notes in Math., vol. 698, Springer-Verlag, Berlin-Heidelberg-New York, 1978, xiv + 182 pp., $9.80.

Many mathematicians, of whom I am one, find orthogonal polynomials fascinating. I was introduced to the Legendre polynomials by O. D. Kellogg, in a course on potential theory, almost half a century ago. At the time, I was entranced more by their elegant formal properties than by their applications. Later, I encountered other orthogonal polynomials. One of the ways in which they arise is as eigenfunctions of differential equations, where the boundary condition is just that of *being* a polynomial, and so involving only finitely many parameters. Perhaps if high-speed computers had been invented earlier, the computational advantages of polynomial solutions would have seemed less compelling, but it is hard to imagine that the so-called classical polynomials (Laguerre, Hermite, Jacobi–Legendre and Chebyshev are special cases) could have escaped notice for long.

I expect (without having actually investigated their history) that all the named systems had been studied by predecessors of the mathematicians they are named for. The Bessel polynomials, however, are exceptional: they appear not to have been studied by Bessel (although they are related to Bessel functions), and were named by Krall and Frink [2] in 1949. They had, in fact, been more or less known at least since 1873, and had occurred in connection with the irrationality of π, statistics, and the wave equation; and were introduced (independently) at about the same time in electrical engineering. Such an ubiquitous set of polynomials surely deserves not only a name but more than the casual mention it got in the Bateman Project volumes [1] in 1953.

The paper by Krall and Frink was actually the first systematic study of the Bessel polynomials; since objects of mathematical discourse, like continents, are so often named for those who popularize them rather than for those who discover them, it is only because of Krall and Frink's good taste that we do not now know these polynomials as the Krall-Frink polynomials. Some of the subsequent active research on Bessel polynomials seems to have been inspired by Krall and Frink's calling attention to the orthogonality of the polynomials–in the complex plane rather than on the real intervals where the classical polynomials are orthogonal.

Grosswald's bibliography lists 116 titles dealing with Bessel polynomials. The book is a quite detailed survey. It describes not only the analytic properties such as one finds for the classical orthogonal polynomials in Szegö's book [3], but also algebraic properties (irreducibility, the Galois group). Grosswald has also provided abstracts of many results that he could not cover in detail; he discusses many applications of Bessel polynomials; calls attention to some unsolved problems about Bessel polynomials; and outlines their history in a preliminary chapter (from which I have borrowed most of the historical remarks in this review).

It is too much to hope that the appearance of this book will prevent the Bessel polynomials from being reinvented, but it will be useful to anyone who comes across them, or one of their variants, and is resourceful enough to find it. Perhaps eventually someone will organize the literature of orthogonal polynomials in inverse form, listing desirable properties and typical problems, and indicating which polynomials have the properties or help solve the problems. Until the arrival of that millennial day, treatises like this one are all we can reasonably expect, and we should be duly grateful to Grosswald for making the Bessel polynomials more accessible.

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The finite element method for elliptic problems, by Philippe G. Ciarlet, North-Holland, Amsterdam, New York, Oxford, 1978, xvii + 530 pp., $56.95.

There is a wide variety of numerical techniques, particularly for the solution of partial differential equations, that go under the heading of finite element methods. The most elementary version of these methods occurs in the context of the Poisson equation where they typically are a special case of the classical variational methods developed by Galerkin, Rayleigh, and Ritz. The latter are based on the Dirichlet Principle which asserts that the solution of the boundary value problem uniquely minimizes a quadratic functional, normally called the energy functional, over a certain class U of functions. The classical idea is to obtain approximations by minimizing the energy functional over a finite-dimensional subspace of U [1], [2]. What distinguishes finite element methods in this context is the particular choice of the finite-dimensional subspace that is used in the approximation. In particular, finite element methods are typically based on spaces of piecewise polynomial functions associated with a simplical decomposition of the region.

There is not general agreement concerning the originator of these ideas although most numerical analysts quote either Courant [3] or Synge [4], both of whom had the basic ideas concerning the elementary mechanics of the method. Later in the mid 1960s engineers independently started an intensive development of the method [5], and several successful applications to large and complicated problems that were originally thought to be intractable generated almost overnight popularity in engineering circles.