should be of value to anyone interested in the development of analysis. I was taught that mathematics was the art of avoiding computation but that adage would have seemed strange to any of the mathematicians mentioned above. Indeed the advent of the electronic digital computer has wrought such a profound change in numerical methods that today's numerical analysis does not seem to owe much to yesterday's efforts. The problems are different, the tools are different, and so are the goals (tables are out, pictures are in).

In any case the character of modern numerical analysis is irrelevant to the value of this book. We should all be grateful to Goldstine (and to IBM) for giving us the opportunity of seeing some great mathematicians at work.

B. N. PARLETT

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The economics of space and time, by Arnold M. Faden, The Iowa State University Press, Ames, Iowa, 1977, xiii + 703 pp., $39.95.

The major vehicle for scholarly activity these days seems to be the journal article. At least, this holds true for scientific work at major American universities. Most scientific books are concerned with surveying, expounding or systematizing work that has had its origin in journal articles.

This is probably an appropriate state of affairs. The fact that most journal articles are more or less carefully refereed helps to screen out uninteresting or erroneous contributions to scientific knowledge. Publication rights are generally guarded with some care by a jury of one's peers.

The publication of Faden's *The economics of space and time* is a distinct departure from the journal article paradigm in at least two respects. First, it is a massive volume (703 pages) of original research, the bulk of which is appearing in print for the first time. Second, this research was undertaken in "splendid isolation"–to use the author's own words–over a period of more than a decade.

This isolation was no doubt due to a large extent to the subject matter of the book: it attempts to apply the mathematics of measure theory to the economics of spatial location. Measure theorists are a subset of mathematicians, and location theorists are a (rather small) subset of economists. The intersection of these two sets is, if not a set of measure zero, at least as close as one can come for all practical purposes.

Despite the difficulties of working in such isolation, I would say that Faden has produced an interesting and readable book. Measure theorists can profitably examine the book for the novel applications of measure theory to economic problems, and spatial economists can examine the book for generalizations and extensions of classical results, as well as some interesting suggestions of new techniques for research in this area.

Faden states at the outset that "... the thesis of this book is that measure theory is the natural language of spatial economics and, indeed, for all social science." The last phrase should perhaps be excused as parental exaggeration, but the author makes a reasonably good case for the first part of his thesis.

Spatial economics is concerned with how economic entities locate themselves in space. Spatial economics, or at least formal models of location, began with the work of von Thünen in 1826. This work stimulated a German school of spatial economists, and to this day it is fair to say that the bulk of our knowledge of spatial economics is due to this German school. Classical location theory probably solidified in the 1930s and since then has been a somewhat neglected branch of economic analysis. The sixties saw a new flourishing of urban economics and regional science and researchers in these areas have, rather naturally, returned to some of the main themes of classical location theory. Armed with a more powerful set of technical and analytic techniques than their predecessors, this new brand of spatial economists may yet revitalize their discipline.

To get a flavor of the kinds of problems that spatial economics examines, let us consider the classical von Thünen problem of location of agricultural activity. We will consider the simplest possible model. We suppose that a central market is located on a homogeneous plane, on which two crops, corn and beans, can be grown, and shipped to the market for eventual consumption. The question is: Which crop will be grown in which locations? There are three kinds of costs relevant to this problem: (1) the prices and production costs of the crops; (ii) the transport costs to the market; and (iii) the land rent costs. The first two costs are taken to be exogenously given while the third cost–the land rents–are to be determined in the course of the solution.

A highly simplistic analysis of the problem goes like this. We let x be the distance to the market, and suppose that y_b bushels of beans can be produced at a cost of k_b per bushel, irrespective of location. The cost of shipping a bushel of beans to market is $c_b x$, and the price of beans is p_b per bushel. The profit from producing beans at x is therefore:

$$R_b(x) = (p_b - k_b)y_b - c_b y_b x.$$

R_b is the maximum amount that a bean farmer will be willing to pay to rent the land at x, and a similar "bid-rent" curve can be derived for a corn farmer which we will denote by $R_c(x)$. Competition between farmers, and greed among rentiers, ensures that the equilibrium rent at each location will be the maximum of $R_b(x)$ and $R_c(x)$. Thus the pattern of land use and rents become clear: we will have two concentric rings of agricultural production; corn or beans will be produced closer to the market as $R_c(0)$ is greater or less than $R_b(0)$; furthermore the rent payed at location x will simply be $R(x) = \max(R_b(x), R_c(x))$.

This model virtually cries out for generalization on several fronts. On the grounds of geographic realism we might consider relaxing the assumptions of the homogeneous plane, the single central market, and the linear transportation costs. It is not hard to see how a system of freeways or railroads may impose a non-Euclidean metric on the agricultural plane–the city block metric $d(x, y) = |x_1 - y_1| + |x_2 - y_2|$ is an obvious example. Such a metric will distort von Thünen's concentric rings, but leave the basic structure of the model intact.

One might also choose to model the land use at locations x in a more explicit way. We have taken x to be a dimensionless point in the plane; but at the same time we have asserted that some positive amount of corn or beans can be grown on this point lot! It is clear that it would be better to think of land productivity (y_b) as being some kind of density function, or going a step further, to think of the yield of a plot of land as being a measure defined on the Borel sets of R^2. From this generalization, it is a minor step to considering the problem on an abstract measure space. The resulting analysis can now be applied to a wide variety of geographic situations: locations of businesses in a city, the seating pattern of an audience at the opera, or even the pattern of use of a mathematicians's desk top.

On the other hand, this generality does not come for free. As soon as one puts a measure space structure on the system being studied, it becomes necessary to verify technical details of measurability and integrability of derived structures, to impose certain perhaps unreasonable linearity assumptions at times, and to insert "almost everywhere" at judiciously chosen places in the conclusions of theorems.

Is it worth it? Do the virtues of added generality and mathematical rigor compensate for the technical difficulties of the proofs? The interested reader will have to judge for himself. In my own case, I can say that Faden has convinced me that for at least *some* applications, measure theory does make a positive net contribution to the economics of the problem.

Having, I hope, given an impression of the kinds of questions addressed by Faden, let me turn now to a brief examination of the contents of the book. The first 100 pages or so are devoted to standard measure theory on abstract measure spaces. The author attempts to motivate the definitions a bit more than is commonly done in mathematics texts, (to his credit I might add), and this section should be accessible to a reader trained in analysis at the undergraduate level. (Proofs of standard measure theory results are generally omitted.)

The author then considers a generalization of the concept of a signed measure which he calls a *pseudomeasure*. A pseudomeasure is a kind of Jordan decomposition where the upper and lower variations may take on values of + or − infinity.

This kind of construct appears to be useful in problems where "gross" quantities may be unbounded but net quantities are more or less well defined. For example consider the problem of maximizing an integral of benefits minus costs over an infinite time horizon. There will generally be a wide variety of programs with unbounded values of benefits and benefits-costs. Economists have resorted to a variety of criteria to choose optimal policies in these cases. One such criterion is von Weizäcker's "overtaking criterion". Clearly one program is better than another if its net benefits eventually exceed the net benefits of the other. Faden uses pesudomeasures to generalize and refine this concept as well as several other proposals which deal with the infinite net benefit problem described above. Showing that these techniques are special cases of a more general construct seems to me to be a real contribution to the literature on optimization problems with an infinite domain.

Having developed the analytical tools to compare unbounded integrals, Faden goes on to apply these tools to general problems of constrained and unconstrained optimization. He presents some results on existence and uniqueness of optimal solutions, and a number of variations on "shadow-price" conditions to characterize constrained optima.

The analytical tools described and developed in the first third of the book are used to analyze various problems of spatial economics in the latter two thirds of the book. There are discussions of the real estate market, the transportation and transhipment problems, several variants on the von Thünen system mentioned earlier in this review, several models of industrial location, and discussions of a number of other topics in spatial economics. These models will no doubt stimulate some fruitful interaction between measure theorists and spatial economists; perhaps the intersection of these two disciplines may yet be of positive measure!

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Obstruction theory on homotopy classification of maps, by Hans J. Baues, Lecture Notes in Math., vol. 628, Springer-Verlag, Berlin, Heidelberg, New York, 1977, xi + 387 pp.

In the beginning was the word, and the word was homology. The great breakthrough in the successful attempt to apply algebraic methods in topology was the discovery of the homology groups and the proof of their topological invariance. Before the homology groups were explicitly described we had the idea of the homology class of a cycle on a manifold or, more generally, a polyhedron, but the description of the collection of homology classes was arithmetical rather than algebraic. That is to say, the great pioneers spoke of Betti numbers and torsion coefficients. It is generally supposed that Emmy Noether was responsible for observing that in fact the homology classes of cycles form an abelian group and that the Betti numbers and torsion coefficients were simply the invariants of finitely generated homology groups. The topological invariance of the homology groups is a truly wonderful result. We define these groups in terms of a very specific and arbitrary combinatorial structure on the topological space and then prove that they are in fact independent of that structure.

Thus algebraic topology was born. Subsequently came the cohomology groups. At first the view was taken that the combinatorial structure on the space gave rise to chain groups and that there were two operators on these chain groups, the boundary operator, or lower boundary operator as it was sometimes called, and the coboundary operator, or upper boundary operator as it was sometimes called. Subsequently it was realized that this was not a good point of view. One should exploit the natural duality to introduce not only chain groups but cochain groups and then one obtained cohomology groups from the cochain groups by a method entirely analogous to that whereby one obtained homology groups from the chain groups. This point of