person who wants to learn commutative harmonic analysis I would recommend several less specialized books instead. And for the student interested in spectral synthesis I would recommend Benedetto's book in conjunction with other books on the subject such as [8], [7], [2], [5] or [1]; incidentally [1] seems better organized than the present text. A lot can also be learned by going back to the original sources, for instance [9].

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The theory of approximate methods and their application to the numerical solution of singular integral equations, by V. V. Ivanov, Noordhoff International Publishing, Schuttersveld 9, P. O. Box 26, Leyden, The Netherlands, xvii + 330 pp., price Dfl. 70,--.

The main theme of the book is the numerical solution of singular integral equations with Cauchy kernels. The following set up is typical. Let γ denote the unit circle in the complex plane and consider the equation:

$$K\phi \equiv K^0\phi + \gamma k\phi = f, \tag{1}$$

where the dominant operator K^0 and the operator k are given by

$$K^0 \equiv a(t)\phi(t) + (\pi i)^{-1}b(t)\int_\gamma \phi(\tau)(\tau - t)^{-1}d\tau, \quad k\phi \equiv \int_\gamma k(\tau, t)\phi(\tau)d\tau,$$

with the first integral having its Cauchy principal value. In the classical theory (cf. [1]) the solution ϕ is sought in the Hölder class $H(\alpha,\gamma)$ $(0 < \alpha \leqslant 1)$, and the coefficient functions together with $(k\phi)(t)$ are assumed to be Hölder continuous on γ. There is also an L_p theory in which these restrictions on the coefficient functions and the kernel $k(\tau, t)$ are relaxed somewhat.

There is a good case and a bad case. The good case is when $a^2 - b^2 \neq 0$ on γ, and bad is "not good".

In the good case, K is normally solvable, there is a simple formula for the index m of (1), namely $m = \arg[(a - b)(a + b)^{-1}]_\gamma/2\pi$, and the classical theory of Noether applies. Also (1) is "regularizable" in the sense that it may be transformed into a Fredholm integral equation of the second kind which is equivalent to (1) (cf. [2], [3]). In the bad case there is no such neat theory; K may not be normally solvable and the problem (1) may be ill-posed.

The good case allows a choice of approaches to the numerical solution of (1). One would be to examine the above regularization procedure, with a view to approximating it numerically. When this is feasible, the problem would be reduced to the numerical solution of a Fredholm integral equation of the second kind, and there are many methods and algorithms for the latter.

The other approach would be to attack the problem (1) directly, and this is the procedure advocated by the author for (1) in the case of a single (scalar) equation. An advantage of the direct approach is its possible application to the solution of (1) in the bad case. In the direct approach, the author (§§10 and 11) first develops methods for the approximate evaluation of singular integrals, as well as for Cauchy integrals based on the constructive theory of functions. Then the methods of least squares and of collocation are used for the solution of (1) in the good case, with sufficiently smooth data, and where γ is not a characteristic value.

However, as the author points out (p.200), in the case of a system of singular integral equations, or when the data is less smooth, other methods such as iteration methods may be more effective. Also, it should be mentioned that for (1) when a,b,k are matrices and f and ϕ are column vectors, the author favors a regularization procedure (when possible) in order to reduce the problem to the solution of system of Fredholm equations of the second kind.

The author treats the subject of equations of convolution type briefly. By means of Fourier transforms and a change of variable effected by a linear fractional substitution, the real axis is mapped onto the unit circle, and a Wiener-Hopf equation of the second kind, for example, is converted into an equation of the form (1) with $k(\tau,t) \equiv 0$ (again a,b,k may be matrices). Alternatively, the Wiener-Hopf equation of the second kind may be solved directly. An example is given of an application of the Galerkin method to an equation of the above type obtained by Grinberg and Fok in their study of plane electromagnetic waves (p. 269).

In giving applications to physics and engineering the author avoids the long known ones to elasticity theory and fluid dynamics (see, e.g., [1]). Instead he treats systems of singular integral equations arising in quantum field theory and in dispersion theory. The most extended discussion of an application occurs in the chapter on equations of convolution type, and is devoted to problems in theoretical electrical engineering. These include the approximate calculation of Fourier transforms, the determination of the dynamic characteristics of linear filters, and the synthesis of optimal linear systems.

The above remarks indicate the content of the portion of the book dealing specifically with singular integral equations. On the other hand, the first chapter, which comprises about forty percent of the book, contains various methods and algorithms for approximating the solutions of operator equations in Hilbert and Banach spaces. The methods are presumably selected so as to be applicable to the numerical solution of singular integral equations in the later chapters. Most of the methods are variational in nature.

The first thing described in the section entitled the "general theory of approximate methods" is a version of Tikhonov's method of regularization for the operator equation (2) $Ax = y$, where $x \in H_1, y \in H_2$, and H_1 and H_2 are Hilbert spaces. Suppose that A is bounded, and assume that A and y are known with an accuracy of ε, i.e., there is a bounded operator A_ε from H_1 into H_2 and an element y_ε of H_2 such that $\|A - A_\varepsilon\| \leqslant \varepsilon$ and $\|y - y_\varepsilon\| \leqslant \varepsilon$. The regularization method begins with the introduction of the quadratic functional to be minimized:

$$(3) \qquad I^{\alpha,\varepsilon}(x) \equiv \|A_\varepsilon x - y_\varepsilon\|^2 + \alpha\|x\|^2,$$

where the nonnegative parameter α may depend on ε. Tikhonov **[4]**, **[5]** used his method for the solution of ill-posed problems such as that of solving a Fredholm equation of the first kind, and proved a convergence theorem, essentially under the conditions (i) the closed unit ball in H_1 is compact in H_2, and (ii) $c_1\varepsilon^2 \leqslant \alpha \leqslant c_2\varepsilon^2$ as $\varepsilon \to 0$.

The author's treatment is a bit different. Without any compactness assumption, he observes that the Euler-Lagrange equation for (3) has the form (4) $(\alpha I + A_\varepsilon^* A_\varepsilon)x = A^* y_\varepsilon$, so that $(\alpha I + A_\varepsilon^* A_\varepsilon)^{-1}$ exists, is bounded, and may be exhibited as a modified Neumann expansion as long as α is positive. The author's convergence theorem for this regularization method is as follows. Let $A^*Ax = A^*y$ be solvable, and let x^* be its unique solution which is orthogonal to the null space of A. Let $x^{(\alpha,\varepsilon)}$ be the solution to the problem of minimizing (3), and let $x_n^{(\alpha,\varepsilon)}$ be a minimizing sequence for (3) such that the quantities α, ε/α, and $\alpha^{-1}\{I^{\alpha,\varepsilon}(x_n^{(\alpha,\varepsilon)}) - I^{\alpha,\varepsilon}(x^{(\alpha,\varepsilon)})\}$ tend uniformly to zero. Then $\|x_n^{\alpha,\varepsilon} - x^*\|$ approaches zero. The case in which A is a closed but unbounded operator is also examined, and a modified version of the above convergence result is obtained.

Later in the chapter, in the discussion of the method of least squares, the author returns to the method of regularization. However, he also does the standard case separately, in which the parameter α in (3) is zero.

It is known that the convergence of Tikhonov's method of regularization may be very slow (see **[6]**). Moreover, as the author indicates (p. 21), effective a priori estimates for α,ε and n as a function of a given deviation δ between the approximate and exact solutions are not known, except in special cases. It is difficult to account for the prominent position of the method in the book, especially since there is sparse use of it in later portions of the book. It is mentioned in connection with the "bad" case in which $a^2 - b^2$ may have isolated zeros on the unit circle (§14) and for systems (§15) etc., but it is not actually applied in either case.

The discussion of the approximate solution of operator equations includes, in the extensive section on the method of least squares, an orthogonalization algorithm, and Kantorovich's steepest descent algorithm. The Ritz-Galerkin method is considered briefly, mostly in its relation to the method of least squares. Iterative methods include simple iteration, for which a thorough discussion is given concerning estimation of the total error for the nonlinear case, and steepest descent methods. Other methods and algorithms, mostly consisting of modification of some of those already mentioned, are briefly stated with citation of references. Chapter 1 ends with an interesting discussion of criteria for the comparison of different algorithms.

The statement of the translation editor that this is the only book in English which deals extensively with the approximate solution of integral equations (with Cauchy kernels) may still be true. At least this reviewer could find no other. However, for the case of convolution equations there is the excellent and extensive work by Gohberg and Fel'dman [7], which forms an effective complement to the present book, since the approach and the methods in the two monographs are quite distinct for the most part.

One feature of the book which aids the reader is to set off passages which can be omitted on a first reading or are suitable "for a reader with an advanced mathematical training", by a vertical line within the margin of the text. The book is "closely written", in the sense of the theorem-proof style, sometimes proceeding without clear motivation from the readers point of view. However this style has the advantage of the presentation of a large amount of material in a short compass, and of clearly setting out what is known and not known. For a book on numerical analysis there are remarkably few numbers included. The only example noticed where any details of calculation were cited was the computer solution of the system which arose in dispersion theory (p.263)

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Nonparametrics: *Statistical methods based on ranks*, by E. L. Lehmann, Holden-Day, Inc., San Francisco, and McGraw-Hill International Book Company, Düsseldorf, Johannesburg, London, Mexico, New York, Panama, São Paulo, Singapore, Sydney, Toronto, 1975, xvi + 457 pp.

The methods taught in the standard basic statistics course assume that the observations come from a normal (Gaussian) distribution. Students are taught that the best estimate of the average or typical value of a population is the