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MARKOV CELL STRUCTURES

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ABSTRACT. We show that the partition underlying a Markov partition for a dynamical system can be chosen to be a cell complex structure.

Let M denote a Riemannian manifold of dimension m, Λ a compact subset of M lying in the interior of M, and $h: M \longrightarrow M$ a diffeomorphism. Recall that Λ is called a *hyperbolic* set for h (see [6]) if

(a) $h: \Lambda \longrightarrow \Lambda$ is a homeomorphism;

(b) $T(M)|_\Lambda$ splits as a direct sum $\xi^u \oplus \xi^s$ of continuous subbundles;

(c) $Dh(\xi^u) = \xi^u$, $Dh(\xi^s) = \xi^s$, Dh is expansive on ξ^u and contractive on ξ^s.

If $\Lambda = M$, then $h: M \longrightarrow M$ is called an Anosov diffeomorphism. It is well known that the bundles ξ^u, ξ^s integrate to give transversal foliations W^u, W^s of M. (See [1].) Locally W^u, W^s decompose M into a cartesian product $\mathbf{R}^k \times \mathbf{R}^l$ where k, l are the dimensions of the leaves in W^u, W^s, and $k + l = m$.

A *cell structure* for W^u, W^s consists of a cell structure C for M, such that each cell $\Delta \in C$ splits as a cartesian product of cells $\Delta = \Delta_u \times \Delta_s$ consistent with the local product structure given M by (W^u, W^s). We further require that if $\Delta \in C$ then each of $\partial\Delta_u \times \partial\Delta_s$, $\Delta_u \times \partial\Delta_s$, $\partial\Delta_u \times \Delta_s$ is a cellular subcomplex of C. Let C^i, j denote the subset of M equal the union of open cells

$$\{\Delta \in C|\dim(\Delta_u) \leqslant i, \dim(\Delta_s) \geqslant j\}.$$

A *Markov cell structure* for an Anosov diffeomorphism $h: M \longrightarrow M$ consists of a cell structure C for (W_u, W_s) satisfying $h^n(C^i, j) \subset C^i, j$ for all i, j and some positive integer n.

THEOREM. *There exist Markov cell structures for every Anosov diffeomorphism.*

REMARKS. (1) A Markov cell structure for $h: M \longrightarrow M$ is also a Markov partition for h, but not vice-versa. The partition sets of M underlying a Markov partition of h, as defined in [5], will generally have nonfinitely generated homology groups.

(2) The theorem generalizes to give a Markov cell structure *near* any hyper-

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bolic set Λ, which intersects Λ in a Markov partition for $h\colon \Lambda \longrightarrow \Lambda$ (as defined in [2]).

(3) In the theorem, C car be cnosen so that the inclusion $C^k, l \subset C$ induces an epimorphism on integral homology groups through dimension k. By applying the constructions of [7] to $h\colon C^k, l \longrightarrow C^k, l$, we obtain a commutative diagram

$$\begin{tikzcd} N \arrow[r, "i"] \arrow[d, "g"'] & C^k, l \arrow[d, "h^n"] \\ N \arrow[r, "i"'] & C^k, l \end{tikzcd}$$

where N is a compact differentiable branched manifold, g is a differentaible expanding endomorphism, and i is a homotopy equivalence. There is a cell structure on N which makes g^s a cellular map for some positive integer s.

(4) Using the setup in the previous remark, we can obtain by a purely geometric construction the dual homology classes associated to the transversal foliations W^u, W^s which have been constructed in [3] by a combination of Markov partition and de Rham theory techniques. We can also prove that these dual classes are uniquely determined by their maximal expanding and minimal contracting properties. This uniqueness had already been proven in [4].

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