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Sums of independent random variables, by V. V. Petrov, Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 82, Springer-Verlag, New York, Heidelberg, Berlin, 1975, x + 345 pp., \$39.60.

Many of the early questions in probability theory concern sums of independent random variables, and in the period 1920–1940 research in probability theory was virtually synonymous with the study of sums of independent random variables. It was during these very active 20 years that some of the most exciting theorems were proved. To name a few, which are nowadays standard classroom material: the three series criterion (Khinchine and Kolmogorov, 1925); the equivalence of convergence of an infinite series in distribution, in probability and with probability one (Lévy, 1937); the strong law of large numbers (Kolmogorov, 1933); the law of the iterated logarithm (Khinchine, 1922; Kolmogorov, 1929, Hartman and Wintner, 1941); the classification of the infinitely divisible laws and conditions for convergence to these laws (Lévy, 1934, 1935; Khinchine, 1937); the Lindeberg-Feller theorem (1922, 1935–1937).

With the above results a plateau had been reached in the study of limit theorems, which had started with de Moivre (1730) and Laplace (1812). Already in the late thirties attention started shifting and probabilists became more interested in Markov chains, continuous time processes, and other situations in which the independence between summands no longer applies, and today the center of gravity of probability theory has moved away from sums of independent random variables. Nevertheless, much work on sums of independent random variables continues to be done, partly for their aesthetic appeal and partly for the technical reason that many limit theorems, even for dependent summands, can be reduced to the case of independent summands by means of various tricks. The best known of these tricks is the use of regeneration points, or "Doeblin's trick". In large part the fascination of the subject is due to the fact that applications and the ingenuity of mathematicians continue to give rise to new questions. E.g. renewal theory was inspired by risk theory for insurance companies, the theory of optimal stopping by sequential analysis, invariance principles and functional limit theorems by examples and the search for a simple proof of the Kolmogorov-Smirnov test, fluctuation theory by queueing theory and surprising combinatorial proofs of limit theorems for maxima. Potential theory for random walks, group valued and Banach space valued random walks, imbedding theorems and rates of imitation of normality, the search for "universal laws" (results which hold for partial sums of all sequences of independent identically distributed random variables), the Erdős-Renyi law of large numbers, and sums of random variables indexed by $\mathbf{Z}^d$, $d > 1$, or other graphs, had a more purely mathematical origin.

In all the years that these newer phenomena were being discovered, people also continued working on more direct generalizations and refinements of the classical limit theorems listed in the first paragraph. Petrov's book is almost exclusively concerned with these refinements. The book stresses the situation where the independent summands are not necessarily identically distributed, an area to which Petrov himself has made significant contributions. After a derivation of the classical limit laws the book gives an excellent survey (up till 1972, the date of the publication of the Russian edition) of error estimates, asymptotic expansions, local limit theorems and large deviation estimates, all for the case of a normal limit law in one dimension. The last two chapters give conditions for the laws of large numbers and the law of the iterated logarithm. Each chapter ends with a supplement, listing (with references) technical results which should be of particular interest to experts. Since much of the recent work in all these areas is not easily accessible to western readers, the book provides a welcome service. Even though the reviewer's preferences are more in the direction of the newer aspects of sums of independent random variables, one should not undervalue the refined versions of the classical limit theorems. Many of the proofs require tremendous skill in classical analysis, especially Fourier analysis, and many of the results, such as the Berry-Esseen estimate, the Edgeworth expansion and asymptotic results for large deviations, are important for statistics and theoretical purposes.

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Mathematical developments arising from Hilbert problems, edited by Felix E. Browder, Proc. Sympos. Pure Math., vol. 28, American Mathematical Society, Providence, Rhode Island, 1976, xii + 628 pp., $37.60.

To begin this review I move a vote of thanks: to the AMS, to the organizing committee of the May, 1974 De Kalb symposium on Hilbert's problems (Bateman (Secretary), Browder (Chairman), Buck, Lewis, Zelinsky), and to all the authors. Admirers of Hilbert and his problems, a set which must be nearly identical with the set of mathematicians, will lovingly place the book on their shelves (as soon as their orders are filled).

Some readers may be curious concerning what was previously available by way of a survey of Hilbert's problems. Since these references do not appear in the present book, I shall furnish them here. Bieberbach [1] surveyed the status of the problems in 1930, as did Demidov [2] in 1966. In 1969 the Russian survey [6] appeared, followed by a translation into German [7]. There is a survey in Japanese by Sin Hitotumatu. Until now the only account in English was that of Fang [3]. There is some biographical information concerning Hilbert's talk on the problems in [8] and [9].

Browder's preface asserts that the main thrust of the symposium was not toward the history of the problems and their current status but rather toward