class on the three-fold [this is (11.8.1) of the text].

$\kappa = 3$. For surfaces of maximal Kodaira dimension the higher pluricanonical mappings are morphisms; it is unknown whether this is the case for three-folds. It is also not known whether deformations of such three-folds still have $\kappa = 3$. One would hope that a moduli space would exist for such three-folds as one does for surfaces of general type; indeed, some work towards this goal has been accomplished now. Again the case of sextic three-folds in $\mathbf{P}^4$ doesn't seem to have been studied.

The author has indeed provided the mathematical community with a valuable manuscript. It could well serve as the basis for independent study or for a seminar; although, for a seminar topic perhaps a detailed look at the classification theory of surfaces would be more profitable. As a reference it serves best as a guide to the literature; although one notable feature is that it includes some new and better proofs of published results.

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Introduction to axiomatic quantum field theory, by N. N. Bogolubov, A. A. Logunov, and I. T. Todorov, Mathematics Physics Monograph, no. 18, W. A. Benjamin, Inc., Reading, Massachusetts, 1975, xxvi + 708 pp., $32.50.

Let us begin with a brief history of why physicists attach great importance to the quantum theory of fields. Dirac, Heisenberg and other great scientists conceived this theory as a synthesis of two extremely fruitful ideas. On the one hand, relativistic quantum mechanics (the Dirac equation) had extended Schrödinger mechanics to predict quantitatively the fine structure of the hydrogen atom spectrum. It also suggested the existence of antimatter. On the other hand, classical field theory (Maxwell's equations for electromagnetism and the Newton-Einstein theory of gravity) provided the theoretical basis for macroscopic physics. The hypothesis of quantum field theory was that

Maxwell's electromagnetic field $F(x)$ is a quantum mechanics observable satisfying Maxwell's equations. Proposed first in the 1930's, this hypothesis eventually led to the discovery of new splittings in the spectrum of hydrogen, first observed by Lamb and Retherford in 1947 (and whose observation was only made possible by the technological developments of that decade). Presently the related experiment to measure the magnetic moment of the electron provides physics' most accurate result: 9 digit agreement between experimental observation and theoretical calculation. Since the early success of this quantum electrodynamics, many physicists have believed that the framework provided by quantum field theory can be adapted to the physics of all elementary particles.

In contrast with these striking implications for physics, the mathematical foundations of quantum field theory eluded understanding for several decades. The appearance of infinities in the calculations alluded to above (necessitating what physicists call renormalization) led many people to abandon the hope of developing a mathematics of relativistic quantum fields. Some workers proposed finding a new formulation in which the mathematics would fit classical ideas. Other workers proposed hard analysis, but very little work was actually done. As is often the case, the study of an interesting scientific problem led to broader mathematical perspectives, in this case, suited to the physics.

The first serious attempts to formulate quantum field theory in a way that meets the standards of both mathematics and of physics began in the 1950's. An early aspect of this work was the proposal of a basic set of assumptions (axioms) for quantum fields. These axioms were working hypotheses for a mathematical framework incorporating the uncontestable bones: quantum mechanics, the Lorentz group, causality, and a stability assumption. The refinement and consequences of these assumptions were the topic of many years' work, and form the major subject of the book under review. This study overlapped and contributed to several areas of analysis, e.g. group representations, several complex variables, operator algebras, and probability theory.

After an extensive introduction concerned with relativistic quantum mechanics and functional analysis, the authors present Wightman's axioms for quantum fields. These three sections cover the background material in some 350 pages, and provide an accessible entry to the subject. The core of the quantum field theory can be found in Parts 4 and 5. In Part 4 we find the theory of scattering, i.e. how the description of particles can be recovered from a quantum field theory. Basically one gives a physical interpretation to the spectrum and spectral multiplicity of the selfadjoint operator H (the Hamiltonian). In the language of physics, this is the study of particles, bound states, scattering and resonances. There are several approaches to scattering, as given in the work of Lehmann, Symanzik, Zimmermann, Haag, Ruelle, Hepp, and Bogolubov. The connections between the work of the various authors is discussed, and a unique feature of this book is the presentation of Bogolubov's theory. In Part 5, the authors extend their discussion of consequences of the axioms to the connection between the spin of a particle and the statistics which it obeys. Here a unique feature is the discussion of "parastatistics" and "infinite component fields".

Part 6 of the book was written for the English translation and covers two main topics: *Local Observables and Constructive Field Theory*. Much thought has been given to the formulation of axioms in terms of local observables (e.g. a lattice of algebras of bounded operators) whose physical interpretation includes all possible experimental measurements which could be carried out in a space (or space-time) region. This section covers much early work of Haag, Kastler and Araki, but covers only a small part of the interesting results on superselection rules established by Haag, Doplicher and Roberts. The second aspect of Part 6 is an introduction to the existence problem for fields satisfying the axioms and to the analysis of detailed properties of solutions to model equations. This "constructive field theory" has been another major focus in the study of quantum fields over the past ten years. The authors devote the final section of their book to a brief but comprehensive survey of this work up to 1971, when their manuscript was completed.

All in all, the book provides a readable introduction to a large area of mathematical physics. In trying to include many things, the authors are occasionally incomplete or sloppy in minor ways. However, the book complements well the older books on axiomatic field theory by Jost and by Streater and Wightman, and a recent review by Streater in *Reports of progress in physics*. A mathematician interested in physics must be willing to learn some of the language and definitions of the physicist. This book is a good place to begin.

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Proof theory, by Gaisi Takeuti, Studies in Logic and the Foundations of Mathematics, vol. 81, North-Holland/American Elsevier, Amsterdam, Oxford, New York, 1975, vii + 372 pp., $35.50.

Takeuti places himself squarely in the line of development of Hilbert and Gentzen, which we begin by retracing. Proof theory was conceived by Hilbert as the means to carry out his grand program to secure the foundations of mathematics by purely mathematical means which were to be of the most elementary and evident kind. The logical structure of mathematical practice had been successfully mirrored within a variety of formal systems S for algebra, geometry, number theory, analysis, and set theory, all logically based in the predicate calculus (the logic of propositional connectives and quantifiers). The consistency of each such S is a combinatorial proposition which may be shown to be equivalent to a number-theoretical statement of the form (for all natural numbers n) $f(n) = g(n)$, where f, g are effectively computable. Hilbert expected to be able to derive such statements using only quantifier-free logic, recursive definitions of functions and proof by induction. Each derivation of a specific numerical statement by these means has a finite model. This differs from the situation where quantifiers are essentially involved in the reasoning; thus even where the variables of S range over natural numbers one may say that application of the reasoning of the predicate calculus in S implicitly involves infinitistic concepts.

To elaborate a bit: Hilbert spoke of the quantifier-free numerical statements as the "real" ones, and of statements involving noncombinatorial concepts or