Along more technical lines, one has such things as catalogues of maximal solvable linear groups over finite fields. Kindred results are also available for nilpotent groups.

Let us turn to an examination of Suprunenko's book. The first chapter is an introduction to permutation groups, and it seems a bit out of place. A few of the ideas in it reappear in the linear group theory, and there is a strong analogy between permutation groups and linear groups. However, these points are not clearly made in the book, and it would probably have been preferable to intersperse this material in the body of the text, as needed. The second chapter contains gross generalities about the general linear group. It is here that a certain unevenness first appears. There is a lengthy and entirely elementary discussion of the matrix representation of a linear transformation. One presumes that potential readers of an advanced text in algebra have no need for such a discussion; but they might profit from a definition of algebraic groups, which are mentioned without explanation later on. The chapter also contains discussions of Dieudonné determinants and the normal subgroups of the full general linear group.

Chapter III is perhaps the most useful in the book, at least as a contribution to the expository literature. It contains Bass' description of the normal subgroups of the stable general linear group over a ring, and the Bass-Lazard-Serre-Mennicke results on the normal subgroups of the general and special linear groups over the integers. The importance of these theorems in algebraic $K$-theory is well understood.

Other chapters deal with reducibility, imprimitivity, solvable linear groups, periodic linear groups and nilpotent linear groups. The material covered here is generally done better in Wehrfritz' book. Many of the results mentioned earlier are either omitted or mentioned without proof. For example, one cannot find proofs of the Auslander-Swan theorem, nor of Tits' theorem in Suprunenko's book. The method of finite approximation is not discussed.

The book has other deficiencies as well. There are no exercises; the translation (done by the Israel Program for Scientific Translations) is adequate, but so stilted in places that it becomes hard to read; the typography and printing are in the muddy style which we have, unfortunately, come to expect in the publications of this Society. The book also contains bits of Russian chauvinism; for instance, the notion of wreath product is attributed to Kaloujnine, even though it was essentially known well over a century ago. In summary, the book is mediocre. The reader or library with a limited budget would do much better to purchase a copy of Wehrfritz.

W. H. GUSTAFSON

BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 83, Number 3, May 1977

*Lecture notes on queueing systems*, by Brian Conolly, John Wiley & Sons, New York, London, Sydney and Toronto, 1975, 176 pp., $9.95.

The theory of queues is a subarea of the field of stochastic models. It deals with the special stochastic processes which arise from the waiting lines made up of randomly arriving items (customers) which require processing by one or more service units. Particularly since 1950, a large number of papers have appeared each year on the subject of queues. By a conservative estimate, some six thousand papers are now in the literature.

The make-up of this vast collection of articles bears a striking resemblance to that of the older material on ordinary differential equations. Relatively minor qualitative changes in the physical models give rise to formally different sets of equations, which can be analyzed by using one of a small number of methodological approaches. As in the study of ordinary differential equations, it has recently become apparent that qualitative and asymptotic results, combined with an algorithmic approach, are much more powerful tools in the solution of real problems than highly complicated analytic solutions, expressed in probability generating functions or integral transforms. By using the theory of weak convergence of stochastic processes, a macroscopic description of complicated queueing systems has become available and provides us with the beginnings of a statistical mechanics of queues. Earlier fears of a continued unwieldy growth of queueing theory without a corresponding increase in methodological depth, can now be laid to rest in view of the vigorous growth of new and powerful techniques.

In view of the profuse literature, of the increasing diversity of its practical applications and the constant state of flux of the recent methodological investigations, it is difficult at this time to write a comprehensive book on the theory of queues. The author has the considerable task of selecting and presenting the approaches, which are least dependent on the special assumptions of the classical queueing models and therefore have the greatest potential for further generalization and for the development of a unified treatment. Apart from introductory textbooks and a handful of major treatises of considerable length, the remaining recent books on queueing theory have dealt with specific areas of applications or primarily with the analytic methods favored by their authors.

In the latter category, a number of volumes have appeared in Lecture Note series, which frequently do not meet the rigorous editorial standards of printed books.

The book under review is based on the material, presented by the author in a seminar course at the Virginia Polytechnic Institute and State University during the Summer of 1969. With 176 pages, it is a brief text as books on queueing theory go.

The first 120 pages deal with the classical elementary queueing models, such as $M/M/1$, $M/G/1$ and $M/G/\infty$. The remaining material, under the general heading of Unconventional Single Server Systems, deals with models investigated by the author and his students. This is specialized material with limited potential for generalization and whose practical importance still needs to be demonstrated.

The former part of the book, which should have been aimed at the nonspecialized reader, leaves much to be desired. The expository style is unsystematic, with lengthy informal comments in the middle of proofs. Much attention goes to matters of terminology, although the author frequently uses unconventional instead of accepted terms. Without giving a lengthy list of examples, the author defines the waiting time of a customer as including the service time of that customer, contrary to accepted practice. The erlang unit is mentioned without a definition and an elementary abelian argument is labeled tauberian.

The shortcomings of this book are not merely stylistic. The proofs of several theorems are inadequate. In discussing the classical result that the output process of an $M/M/1$ queue is Poisson, the author shows only that the times between three successive departures are independent and negative exponentially distributed. This is not only insufficient, but the author's subsequent statement that the theorem is not valid for more general systems, is incorrect. P. J. Burke's theorem was indeed proved for the $M/M/s$ queue.

When there is a choice of several classical arguments the author has a propensity for selecting the least informative approach as in his presentation of the $M/G/1$ model. A number of formulas are poorly aligned and a lengthy proof ends in mid-sentence on p. 135. Apart from all other considerations, the book would have benefited from greater editorial care.

In summary, except as an accessible reference to the author's own research, this book cannot be recommended as reading material on the classical queueing models. This is unfortunate. There is a definite need for clear and unified expositions of the theory of queues, which provide a broad synthesis of an interesting but overly ramified field.

MARCEL F. NEUTS

BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 83, Number 3, May 1977

*Optimization theory, the finite dimensional case*, by Magnus R. Hestenes, John Wiley and Sons, New York, London, Sydney, Toronto, 1975, xiii + 447 pp., $24.95.

The central problem of *nonlinear programming*, which is one of the four or five main areas within *mathematical programming*, can be stated as follows:

$$(\mathrm{P})\qquad \begin{array}{ll} \text{infimize} & f_0(x) \\ \text{subject to} & f_i(x) \leqslant 0, \quad i = 1, \ldots, r, \\ & g_i(x) = 0, \quad i = 1, \ldots, s, \\ & \qquad x \in S. \end{array}$$

In (P), the functions $f_i$, $g_i$ map $S \subseteq R^n$ to $R$, $x = (x_1, \ldots, x_n) \in R^n$, and typically $S = R^n$ or $S$ is a compact, or convex, or an open subset of $R^n$.

When $r = 0$, i.e., when all constraints in (P) are equalities, and suitable differentiability conditions are imposed, (P) becomes the calculus optimization problem that is a standard topic of most two-semester calculus sequences. Indeed, much of the work in nonlinear programming, which is not aimed at obtaining specific algorithms, is a continuation of classical investigations.

**1. A concept of abstract duality.** A new "twist" on (P) is provided by the fairly recent generalized dual problems, which can be abstractly formulated as follows, with $S \neq \varnothing$ an arbitrary set.