

the theory of rays and associated variational principles. Rays are uniquely valuable in the asymptotic investigation of both linear and nonlinear waves.

It is therefore natural on opening a book on Asymptotic Wave Theory to enquire what aspects of rays are covered. The reader will seek in vain, for there is not a single mention of ray. He will find the Laplace transform, Bessel functions and the method of steepest descent for integrals described and then applied to various problems in water waves and seismology. But the nearest he will get to a ray is in one short section on characteristics. The absence of one of the most powerful modern tools for evaluating the asymptotic performance in many different physical contexts is a serious deficiency in a book purporting to deal with asymptotic theory. Whatever other methods are eliminated in the process of selection this one must not be discarded. The applied mathematician of today dare not be ignorant of rays which can offer a viable approach both analytically and numerically when other techniques are hopeless. The book at his elbow and the book he shows his students need to tell the reader about the propagation of energy along rays, transport equations and Hamilton's principle. Without these topics the value of a book is that much less. Caveat emptor.

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BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 83, Number 3, May 1977

Stochastic processes in queueing theory, by A. A. Borovkov, Applications of Mathematics, no. 4, Springer-Verlag, New York, Heidelberg, Berlin, 1976, xi + 280 pp., \$29.80.

Queueing theory is that branch of applied mathematics which attempts to construct and analyse models for what might be called 'unpredictable congestion'. There are many practical situations in which 'customers' demand some sort of 'service' which they cannot immediately obtain because of the demands of other customers. Very often the congestion is caused by variability, in the arrival pattern of the customers, or in the service mechanism, or both, and any model must be expressed in terms of random processes, and can be expected to yield conclusions in probabilistic language.

The early development of the theory was motivated by the problems of congestion in telephone systems, first in Scandinavia (A. K. Erlang) and later in the United States and France (F. Pollaczek). At first it grew in isolation from other manifestations of applied probability, but gradually the connections with the growing theory of random processes came to be realised and exploited. In the West this process may be said to have been completed in 1951 when D. G. Kendall addressed a famous meeting of the Royal Statistical Society, but in Russia the work of A. Ya. Hincin had by then already introduced the subject to the thriving Russian school of probabilists.

It must be admitted that the last quarter-century has been more notable for quantity than for quality of published research. It is too easy to devise a slightly different queueing system and to study it by what are now standard methods. If one's results can be kept safely under cover of several Laplace transforms, they are safe from comparison with reality. And indeed, those

who have to cope with congestion in the real world often find simulation, or numerical evaluation, to be more useful tools in the computer age.

But blind simulation is a very expensive business, and the mathematician is still needed to provide insights rather than formulae. Information of a qualitative or approximate kind about stability, robustness, sensitivity, rates of convergence, may be just what the practical man needs, or if not will enable him to ask the right questions of his computer. There has been distinguished work along these lines, but much remains to be done; in particular the many-server queue still presents a formidable challenge.

Although the Russian school has been by no means isolated from Western developments, it has inevitably differed in emphasis, and an account from that viewpoint of the present state of the theory is valuable and stimulating. Borovkov has taken a very fundamental approach, fitting a wide variety of models into a general framework. Explicit formulae are kept in their place, and he usefully stresses the limiting results which justify robust approximations of real practical use.

He does not discuss the relevance of the theory to the real world, and the book is only (!) an authoritative synthesis of the underlying mathematics. It will be read, and with great profit, by mathematicians seeking uses for the powerful tools of random process theory. Will they be able to make any contribution, however indirect, to the world which does not read the learned journals? There is no doubt that modern telecommunications systems work better because of the achievements of Erlang and his successors, but some other applications of the theory have been less fruitful (largely because a queue is often a complex feedback mechanism). Perhaps one rather trite conclusion is that here, as in other areas of applied mathematics, mathematicians should direct their attention to questions to which someone, somewhere, wants to know the answer.

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BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 83, Number 3, May 1977

Matrix groups, by D. A. Suprunenko, Translations of Mathematical Monographs, vol. 45, American Mathematical Society, Providence, Rhode Island, 1976, viii + 252 pp., \$31.20.

If I recall correctly, John Thompson prefaced his talk at the group theory symposium at the University of Illinois in 1967 with the remark "I believe in a heliocentric view of the universe, with the linear groups playing the role of the sun". Given the nature of the development of group theory in the past one hundred years, such a credo seems very appropriate. The earliest work in group theory was concerned mainly with permutation groups, the principal example of which was the Galois group acting on the roots of a polynomial equation. Interest in linear groups arose first in a geometric context. Boole and Cayley in the 1840's and 1850's initiated invariant theory, i.e. the study of rational functions of several variables that are invariant under various groups of linear or affine changes of variables. This work occupied the attention of many prominent mathematicians for many years; one major development was the announcement of Klein's Erlanger Program, which stressed that geometric