

particular book) it is simply a mistake to try to teach this material to students without some background in abstract mathematics. They can not appreciate what is going on. For students *with* such a background, this could be a useful book, but it should be supplemented with many, many more examples and applications. Of course this is exactly the kind of thing a teacher can and should do.

**4. The Ponasse book.** This book concentrates almost exclusively on the Completeness Theorem and its relation with Boolean algebra and general topology. The Compactness and Löwenheim-Skolem Theorems are there, but never discussed or used. There are almost no examples or applications, and the Incompleteness Theorems are not mentioned. What the book *does* cover could be useful to a student of logic, but it would be an unfortunate way to introduce mathematics students to the basic concerns of mathematical logic.

JON BARWISE

BULLETIN OF THE  
AMERICAN MATHEMATICAL SOCIETY  
Volume 83, Number 2, March 1977

*Introduction to spectral theory: Selfadjoint ordinary differential operators*, by B. M. Levitan and I. S. Sargsjan, Translations of Mathematical Monographs, vol. 39, Amer. Math. Soc., Providence, Rhode Island, 1975, xi + 525 pp. \$52.80.

This book gives a systematic account of a number of basic topics in the modern spectral theory of selfadjoint ordinary differential operators, particularly second order and a system of two first order operators. It also contains, in substantially less detail, the spectral theory concerning  $n$ th order operators and is simply meant to serve as an introduction to their area of study.

A differential operator is said to be regular if the domain of its variables is finite and its coefficients are continuous. If the domain is infinite and/or all or some of the coefficients are not summable, then the differential operator is called singular. The basic spectral theory of regular second order differential operators consists of the Sturm-Liouville theory and much space is devoted in this book to regular problems. Nonetheless, the principal content of the book is the spectral theory of singular operators. This theory was founded by H. Weyl whose work, together with the classical moment problem, played an important role in the development of a general spectral theory of symmetric and selfadjoint operators, through the efforts of F. Riesz, J. Von Neumann and others. H. Weyl's remarkable result on the limit circle and limit point gives a complete description for symmetric second order differential operators and of all its selfadjoint extensions. The general problem of describing all selfadjoint extensions of a symmetric operator was solved later by J. Von Neumann. A large role in popularizing the spectral theory of differential operators was played by the monographs of E. C. Titchmarsh, in which a new approach to the theory of singular second order operators was given. Much space is allotted in this book to singular systems of two first order operators also.

Although it appeared at the beginning that the abstract spectral theory

completely covers the various special cases, it gradually became clear that it gives for many important questions either no answers at all or very inadequate answers. For example, this is the case concerning the asymptotic behavior of eigenvalues and eigenfunctions. Moreover, one of the basic theorems of the entire abstract theory is the spectral decomposition of an operator in terms of a so-called resolution of the identity. For differential operators this spectral decomposition can usually be described by means of the solution of an appropriate equation. Thus in questions regarding the specific description of the resolution of identity, the general spectral theory is of very little help. This perhaps explains the paradoxical fact that a few years after the actual completion of the abstract spectral theory of selfadjoint operators, intensive work on the spectral theory of selfadjoint differential operators began.

For the convenience of the reader who is not familiar with abstract spectral theory, Chapter 13 discusses concisely this theory without proofs and indicates various connections with the spectral theory of differential operators. Also necessary results from analysis that are used in this book are given in Chapter 14 as a ready reference.

The material of the book is very well organized and the proofs are clear. It is an excellent source for anyone who wants to learn or review the essentials of the subject. It is a valuable and welcome addition to the literature.

V. LAKSHMIKANTHAM

BULLETIN OF THE  
AMERICAN MATHEMATICAL SOCIETY  
Volume 83, Number 2, March 1977

*Logic in algebraic form*, by William Craig, Studies in Logic and the Foundations of Mathematics, vol. 72, North-Holland Publishing Company, Amsterdam and London, 1974, 204 + viii pp., \$18.50.

*An algebraic approach to non-classical logics*, by Helena Rasiowa, Studies in Logic and the Foundations of Mathematics, vol. 78, North-Holland Publishing Company, Amsterdam (also, PWN-Polish Scientific Publishers, Warszawa) 403 + xv pp., 1974.

The two volumes under review belong in spirit to the classical period of algebraic logic which is the study of logical problems by algebraic means and of algebraic structures arising in mathematical logic. They do not use the tools of categorical logic which is the modern inheritor of the subject. They are concerned with propositional and first order languages, theories expressed in them and algebraic structures derived from them.

Craig's approach to algebraic logic is highly personal and, although it links up with the theory of polyadic and cylindric algebras, it has its own orientation and very distinctive flavour. *Logic in algebraic form* is thought provoking and worth studying from the philosophical, proof-theoretic and algebraic viewpoints. Should such formulas as  $P(v_0)$  and  $(v_1 = v_1) \wedge P(v_0)$  be identified in an algebraic formulation of first order logic? They are, of course, provably equivalent and so would be identified in the cylindric or polyadic algebra setting. They are not identified, say, in Lawvere's categorical logic approach to elementary theories. Craig argues that they should not because they do not determine the same operation, that of the first formula depending