so far have been of limited applicability. This notwithstanding, they are the methods used in $SL_2(\mathbf{R})$. The simplest starts from the Poisson summation formula and has, in various guises, been with us for a long time. It works well for subgroups of SL(2, **Z**), and perhaps for subgroups of SL(n, **Z**) too. Its limitations are recognized, and Lang employs it only for the sake of a quick introduction.

The other method is newer and appeared only after the problem of the analytic continuation of Eisenstein series had been solved for general groups. It has exercised a strange attraction on a number of mathematicians, Lang among them, and acquired somehow a reputation of being more analytic. It is in fact not unrelated to Selberg's method, but this flows easily along a natural course, while that moves through a channel cut by the machinery of perturbation or, more precisely, scattering theory. Scattering theory, to which Faddeev has written an enlightening introduction (translated in J. Mathematical Phys., 1963), is important for its own sake, and may be a useful weapon for the number-theorist, and the rest of us too; if not for use against the Eisenstein series which have, after all, already surrendered, then against stronger, more stubborn foes; so we can be grateful to Lang for pressing it into our hands. Moreover, since it has been easy to forget that, like everything else, Selberg's method had antecedents, it is instructive to place it alongside the methods arising from scattering theory and to note the fraternal likeness. But this is of interest only to the initiated. The beginner should be shown an easy path, free of red herring and leading to some outstanding problems, which for the Eisenstein series are usually in higher dimensions and primarily arithmetic, concerned not with the analytic continuation which is known but with the location of the poles contributing to the spectrum. Their solution probably demands a better understanding of the Euler products associated to automorphic forms and of the intertwining operators and their normalizations.

But we should not forget the purpose of the book, which was not intended to teach the reader everything about SL(2, **R**). It is written by an outsider, although not to mathematics or to exposition, for outsiders, and in consulting his own needs he has probably met theirs. $SL_2(\mathbf{R})$, which introduces the harmonic analysis through the Plancherel formula and the analytic theory of automorphic forms through the Eisenstein series, may take its place alongside the author's other books, which for many of us have been the entrance to topics that could otherwise have remained inaccessible.

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The heat equation, by D. V. Widder, Academic Press, New York, San Francisco, and London, 1975, xiv + 267 pp., $22.50.

The title implies that this is a book about a partial differential equation, and so it is; but it is very different from other books about partial differential equations. Older books used to concentrate on more or less explicit solutions of boundary value problems or, as we are more apt to say nowadays, on algorithms for calculating solutions. Modern books are more likely to concentrate on existence and uniqueness theorems in the most general settings possible, and on qualitative information about the solutions that can be discovered without actually producing them. There is one equation, however, that is customarily discussed in a different spirit: the Laplace equation in two dimensions. Here we know next to everything about the existence and uniqueness of solutions, but we also know a great deal about their special properties in special situations. We call the resulting body of theory complex analysis and write separate books about it, possibly without mentioning Laplace's equation explicitly at all. That complex analysis is an independent theory comes about partly because of its many applications besides solving Laplace's equation, but also because the theory has turned out to contain many results that have the elegance and unexpectedness that are characteristic of the best mathematics.

Widder's book presents (for the first time in book form) the principal results of a theory that treats solutions of the heat equation $u_{xx} = u_t$ in much the same way that a book on complex analysis treats analytic functions. Widder presents the theory on its own merits, but near the end of the book gives an extensive table of the analogies between analytic functions and solutions of the heat equation, analogies that either guided, or could have guided, the development of the theory. Analogies between elliptic and parabolic differential equations have recently served as a guide in much more general situations, but here the equation is the special equation for one space dimension, the domain in which solutions are considered is usually an (x, t)-rectangle (possibly infinite), and the theory corresponds most closely to what complex analysis would be if we began with real-analytic functions in an interval and considered their extensions to disks in the plane. The special nature of the theory is mirrored in the precision and elegance of the results that can be established in it. The simplified model of a physical situation that gave rise to the heat equation is discussed in an introductory chapter but then recedes far into the background, and we can watch the theory of temperature functions (solutions of the heat equation) take on a life of its own. It is interesting that the author's work on temperature functions seems, according to his recollections, to have been originally motivated by analogies with harmonic functions rather than by a desire to solve physical problems, although he traces his interest in the subject to a course in mathematical physics given by Hille. Nevertheless some, at least, of the theory certainly has physical content. Widder's integral representation for positive temperature functions, first published in 1944, leads to a uniqueness theorem that is satisfying on physical grounds (absolute temperatures being inherently nonnegative), but has not yet made its way into very many textbooks (although the effect of positivity plays a prominent role in more advanced treatises).

Analytic functions are represented by Cauchy's formula, which can be looked at as convolution with the simplest analytic function with an isolated singularity, namely $1/z$. Correspondingly, temperature functions can often be represented by convolution with the source solution

$$k(x, t) = \exp\left(-x^2/(4t)\right)(4\pi t)^{-1/2},$$

as well as by other integral transforms. Instead of the polynomials z^n we have the heat polynomials $v_n(x, t)$ generated by

$$\exp(xz + tz^2) = \sum_{n=0}^{\infty} v_n(x, t)z^n/n!;$$

other series of polynomials are also useful, just as series of polynomials other than $\{z^n\}$ are important in complex analysis. Temperature functions possess a maximum principle, a reflection principle, and uniqueness theorems showing how they are determined by various kinds of data; there is even an analogue of Liouville's theorem. For a suitably restricted subclass of temperature functions there is Huygens' principle (which gets its name from a quite different analogous theory, optics, i.e. the theory of the wave equation); this says that the values of the function for some t can be used as initial data for determining the function at later values of t, in much the same way that we can take the values of an analytic function on a contour and use them in Cauchy's formula to calculate the function inside the contour. (Poisson's formula for harmonic functions is perhaps a closer analogue.) The analogy between positive temperature functions and positive harmonic functions has already been mentioned. One chapter is devoted to the use of Jacobian theta functions for solving the heat equation in a finite x-interval; the occurrence of these functions is less surprising than one might think, since the theta functions are series of functions $k(x, t)$ or $k_x(x, t)$; they also occur in the construction of the Green's function for an (x, t)-rectangle. One chapter indicates some possible generalizations to higher dimensions; another discusses homogeneous temperature functions ($u(\lambda x, \lambda^2 t) = \lambda^n u(x, t)$). A final chapter considers several special topics.

The book is written in the author's customary polished but condensed style. Much of it consists of simplified versions of his own previous work. The results seem, generally speaking, to be more difficult than their analogues in complex analysis; I do not know whether this is because the latter theory is longer established or because problems about the heat equation are inherently more difficult than problems for Laplace's equation (as suggested to me by A. Friedman). It seems likely, however, that many additional interesting results are waiting to be discovered (or invented, depending on our philosophy of mathematics). Anyone who wishes to participate in the search should have this book at hand.

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Continuous flows in the plane, by Anatole Beck, Die Grundlehren der Mathematischen Wissenschaften in Einzeldarstellungen, Band 201, Springer-Verlag, New York, Heidelberg, Berlin, 1974, x + 462 pp.,$46.80.

A *flow* in a space X is a (continuous) group action of the real line on X; that is, a continuous function φ: $\mathbf{R} \times X \to X$ such that $\varphi(t + s, x) = \varphi(t, \varphi(s, x))$. Behind this simple analytic veil lies, in the case where X is the plane $\mathbf{R}^2$ (or the two-sphere S^2), a beautiful geometric theory. The plane becomes a patchwork quilt. The patches come in infinitely varied and intriguing patterns, that, nevertheless, admit to a surprising amount of classification.