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ZETA FUNCTIONS AND THEIR ASYMPTOTIC EXPANSIONS FOR COMPACT LOCALLY SYMMETRIC SPACES OF NEGATIVE CURVATURE

BY ROBERT S. CAHN AND JOSEPH A. WOLF

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Let G be a noncompact, connected, semisimple Lie group with maximal compact subgroup K. Let Γ be a discrete, cocompact subgroup of G with no nontrivial elements of finite order and denote by M the space $\Gamma\backslash G/K$. M will be a Riemannian manifold with metric arising from the Cartan-Killing form of the Lie algebra of G. The Laplacian of M will have eigenvalues $0 = \lambda_0 \leqslant \lambda_1 \leqslant \lambda_2 \leqslant \cdots$ Let $\zeta_M(t) = \Sigma_0^\infty e^{-\lambda_n t}$. It is standard that

$$\zeta_M(t) \cong (4\pi t)^{-\dim(M)/2}(a_0 + a_1 t + \cdots + a_n t^n + O(t^{n+1})), \qquad t \downarrow 0.$$

Let $M' = G'/K$ be the compact dual of G/K. Then

$$\zeta_{M'}(t) \cong (4\pi t)^{-\dim(M)/2}(a'_0 + a'_1 t + \cdots + a'_n t^n + O(t^{n+1})), \qquad t \downarrow 0$$

and the coefficients a'_n have been computed (see [1] and [2]).

THEOREM. $a_n = (-1)^n (\mathrm{Vol}(M)/\mathrm{Vol}(M'))a'_n$.

"Nolan Wallach informs us that Mr. Miatello has proved this result for symmetric spaces of rank 1 using different methods."

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DEPARTMENT OF MATHEMATICS, UNIVERSITY OF MIAMI, CORAL GABLES, FLORIDA 33124

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF CALIFORNIA, BERKELEY, CALIFORNIA 94720

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