

The authors' most significant achievement is the *systematic* application of ideas of elementary category theory to systems theory; not that this approach to systems study is new. E.g., such ideas are used by Eilenberg (*Automata, languages and machines*, Academic Press, 1975), Arbib and Manes (*Arrows, Structures, and Functors*, Academic Press, 1975), Dididze (Russian), and Bautor (German) for the study of logical systems. The authors of the monograph under review provide elegant, if not tightly reasoned, arguments in this area. On the other hand, there is an undesirable trend in this kind of enterprise. Researchers seem to insist on the development of category-theoretic abstractions of the notion of recursivity rather than developing a "recursive category theory" which begins with a recursively enumerable class of objects together with various recursive functors. Use of recursion-theoretic ideas is not likely to enfeeble the wings of that soaring eagle, category theory. In any case, Von Neumann has warned us all about the adverse things that can happen to the fabric of the mathematical sciences as our theories, governed by aesthetical desiderata alone, recede further and further from contact with physical reality.

In conclusion, this fine work is only a first try at the much desired paradigm for systems theory. Axiomatization is never achieved and much in the overview is left unanswered (e.g., (1) how is the structure of nonlinear systems clarified and extended by the authors' approach, and (2) what *new* systems, if any, are predicted by the present approach). Nevertheless, this carefully written and attractively reproduced treatise passes the BUNTSI test for scientific publishability: much of the material is *Beautiful, Useful, New, True, Serious, and Interesting*.

ALBERT A. MULLIN

An introduction to invariant imbedding, by R. Bellman and G. M. Wing, John Wiley & Sons, New York, 1975, xv+250 pp., \$18.95.

This book gives an introduction at an elementary level to a method for solving boundary value problems in one independent variable. The method is called invariant imbedding and sometimes it is also referred to as the method of continuation. The idea is to let the basic interval, over which the solution is defined, vary and replace the boundary value problem by an initial value problem with the width of the interval as independent variable. For analytical, as well as computational reasons, the initial value problem that ensues is more convenient. However, even linear boundary value problems lead to corresponding nonlinear initial value problems which have the form of Riccati equations.

The idea described above frequently has a clear physical interpretation and the quantity that satisfies the initial value problem has physical significance; for example, it may be the reflection coefficient in transport or wave processes. It was first used by Stokes (1862) in a somewhat crude discrete

form and later by Schmidt (1907) in connection with reflection and transmission of light through inhomogeneous media. Ambarzumian (1943) articulated the approach more clearly and exploited it further, but it was Chandrasekhar who, in his well-known book *Radiative transfer* (Dover, New York, 1960, first published in 1950) stated in full generality the idea described above and called it "principles of invariance". Chandrasekhar proceeded in his book with the exploitation of the principles of invariance extracting a phenomenal amount of information and solving several problems in transport theory that were considered impossible to solve up to the late 1940's. In an interesting article (*J. Mathematical Phys.* **41** (1962), 1-41), R. M. Redheffer gives a lucid survey of some of the early developments, as well as his own contributions, up to 1962. The authors of the present book have participated vigorously in the exploitation of the imbedding idea in a variety of contexts and have written several books and papers on it and its applications in addition to the present one.

The first four chapters explain in a very simple context the basic idea of imbedding and the mechanics of converting boundary value problems to initial value problems. The remaining eight chapters deal with more specific applications such as random walks, wave propagation, calculation of eigenvalues, WKB and integral equations. Many of the devices and techniques presented in this book may be known to a lot of mathematicians, perhaps by different names (or no names at all). The purpose of the book is, however, to reach a wide audience and this will no doubt be accomplished because the writing is lucid and with a lot of redundancy to make reading easy.

Nevertheless it seems that too much space is occupied with generalities while not enough substantial examples are treated. This should be contrasted with Chandrasekhar's book where the general ideas take up a few pages and nearly half the book is devoted to detailed analysis and computations.

Mathematicians may find interesting, however, the several possible applications of the imbedding idea and, one never knows, it may be useful in their own research also.

GEORGE C. PAPANICOLAOU

Cohomology theories for compact abelian groups, by K. H. Hofmann and P. S. Mostert, Springer-Verlag, New York, 1973, 236 pp., \$18.50.

Every compact abelian group is a projective limit of compact abelian Lie groups, each of which is the product of a torus and a finite group. That is, one obtains the most general compact abelian group by starting with the circle group and the finite cyclic groups, then taking products and limits. Thus, in studying invariants for compact abelian groups one should often be able to compute an invariant in general once one knows it for these simple building blocks and understands how it is treated by the product and limit operations.