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THE PRODUCTS OF MANIFOLDS WITH THE f.p.p. NEED NOT HAVE THE f.p.p.

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In [1] Bredon showed that the complex $X_\alpha = S^k \cup_\alpha D^{2m}$ has the fixed-point property with $[\alpha] \in \pi_{2m-1}(S^k)$ being nontrivial, provided that the following condition holds.

CONDITION (*). *k is odd, and $r = 2m - k - 1 < k - 1$.*
But $X_\alpha \times X_\alpha$ admits a fixed-point free map if p, the order of $[\alpha]$, is relatively prime to p', the order of $[\alpha']$. To show that the analogous situation holds for manifolds, let M_{2m} be a $2n$-dimensional compact smooth manifold, with $2m < n$ and $\pi_1(\partial M_{2m}) = \{1\}$, of the same homotopy type as X_α, and put $M = M_{2m} \cup_h M_{2m}$ where $h: \partial M_{2m} \longrightarrow \partial M_{2m}$ is a diffeomorphism.

THEOREM 1. *Suppose in addition to Condition (*) that r is not of the form $2^s - 2$, and that p, the order of $[\alpha]$ in $\pi_{2m-1}(S^k)$ is greater than 2 if $r = 0 \bmod 8$. Then the connected sum $M \# CP^n$, of M and the complex n-projective space CP^n, has the fixed-point property if $n + 1$ is relatively prime to both p and $\varphi(p)$ where $\varphi(p)$ is the Euler function of p.*

To prove the theorem one shows that the Lefschetz number $L(f)$ of any map $f: M \# CP^n \longrightarrow M \# CP^n$ is given by the equation

$$L(f) = -(\kappa + \kappa') + (\mu + \mu') + (1 + \lambda + \cdots + \lambda^n)$$

where $\kappa, \kappa', \mu, \mu'$ and λ are integers such that

$$\kappa\kappa' = \lambda^n = \mu\mu', \qquad \kappa = \mu \bmod q \text{ and } \kappa' = \mu' \bmod q$$

with q being a proper divisor of p. In fact q is the order of the class of $[\alpha]$ in $\Pi_r(S)/\text{image } J$, where $\Pi_r(S)$ is the stable r-stem $\pi_{r+*}(S^*)$ and J the stable J-homomorphism $\pi_r(SO) \longrightarrow \Pi_r(S)$, and the conditions on r are required to ensure that $q > 1$ and that the congruence $\kappa' = \mu' \bmod q$ holds.

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THEOREM 2. *With the assumptions of Theorem* 1, *suppose that* M *and* M' *are the doubles, respectively, of* M_{2m} *and* M'_{2m}. *Then* $(M \# CP^n) \times (M' \# CP^n)$ *does not have the fixed-point property if* $(p, p') = 1$.

To prove Theorem 2 one first retracts $(M \# CP^n) \times (M' \# CP^n)$ onto $M_{2m} \times M'_{2m}$, and then one proceeds to retract $M_{2m} \times M'_{2m}$, according to [1], onto S^k considered a submanifold of the diagonal of $(M \# CP^n) \times (M' \# CP^n)$.

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REFERENCE

1. G. Bredon, *Some examples for the fixed-point property*, Pacific J. Math 38 (1971), 571–575.

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