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CHARACTERISTIC NUMBERS OF UNITARY TORUS-MANIFOLDS

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1. **Introduction.** Unitary torus-manifolds have been studied by Hamrick and Ossa in [5]. They show that the bordism class of such a manifold is determined by its fixed point set. In [4] tom Dieck introduced homotopical bordism theories. Now the question arises: Is a result corresponding to the one of Hamrick and Ossa true for homotopical bordism? The answer is given in Proposition 2.3. From this proposition we get the following

THEOREM. *Unitary torus-manifolds are determined by K-theory characteristic numbers*

The details of all proofs are contained in [6], the author's thesis, which was written under T. tom Dieck.

2. **Characteristic numbers.** There are two ways of defining what an equivariant unitary G-manifold should be. The first is given by stabilizing the tangent bundle with $\boldsymbol{R}^n$ (trivial G-action), the second by stabilizing with complex representations. We denote the bordism theories so obtained by $\bar{\mathfrak{U}}_*^G$ and $\mathfrak{U}_*^G$, respectively. There is an obvious natural transformation $j:\bar{\mathfrak{U}}_*^G \to \mathfrak{U}_*^G$ of equivariant homology theories.

LEMMA 2.1. *j is a monomorphism for compact abelian Lie groups.*

In [4] a homotopical bordism theory is defined, using equivariant Thom spectra. There exists a Pontrjagin-Thom construction $i:\mathfrak{U}_*^G \to U_*^G$.

PROPOSITION 2.2. *If G is a compact abelian Lie group, then i is injective.*

Let S denote the multiplicatively closed set in U_*^G generated by the Euler classes of finite dimensional complex representations. Let $\lambda: U_*^G \to S^{-1}U_*^G$ denote the localization map. As forming $S^{-1}U_*^G$ corresponds to "restriction to the fixed point set" we have in analogy to [5].

PROPOSITION 2.3. *λ is injective iff G is a torus.*

Let EG denote a free contractible G-space, $BG = EG/G$, the projection

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$EG \to$ point induces in unitary cobordism

$$\alpha : U_G^* \to U_G^*(EG) \cong U^*(BG);$$

α is in fact a natural transformation of equivariant cohomology theories [**4**].

PROPOSITION 2.4. *α is injective for a torus.*

PROOF. By using Proposition 2.3 one only has to show that $S^{-1}\alpha$ is injective. This can be reduced to showing this for a group of the form $\boldsymbol{Z}_p$, p a prime. This was proved in [**3**].

Let $p: U_G^* \to U^*$ be "forgetting the G-action". We denote the kernel by I_G. In analogy to [**1**] we have

PROPOSITION 2.5. *Let G be a compact abelian Lie group or a group admitting a free complex representation. Then α induces an isomorphism*

$$\hat{\alpha} : (U_G^*)^\wedge \xrightarrow{\cong} U^*(BG)$$

where $\hat{}$ denotes I_G-adic completion, provided $U_G^{\mathrm{odd}} = 0$.

Let us denote by $B: U^*(X) \to K^*(X)[a_1, a_2, \ldots]$ the Boardman map [**2**]. This is known to be split injective for X a point. This gives

LEMMA 2.6. *The map $B: U^*(BG) \to K^*(BG)[[a_1, a_2, \ldots]]$ is injective for G a torus.*

If we look at the composition $B \circ \alpha \circ i$ then 2.2, 2.4 and 2.6 lead to

THEOREM 2.7. *K-theory characteristic numbers determine the bordism class of a unitary torus manifold.*

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