for the specialist to have the theorems stated in their full generality and to be able to find computations as complete as those given in §29, but for the less experienced it would have been easier if certain results had been obtained as directly as possible without exploring "all the interesting byways."

Nevertheless, the reading of this treatise, no matter how difficult, will always be a rewarding experience for anybody interested in abstract harmonic analysis.

ALESSANDRO FIGÀ-TALAMANCA

Introduction to the Theory of Partially Ordered Spaces by B. Z. Vulikh, translated by Leo F. Boron. Gordon and Breach, New York, 1967. xv + 387 pp.

This title first appeared in Russian in 1961; the present translation appeared in print in 1967, and the reviewer's copy appeared in his mail in 1971. There is little difference perceptible between the reviewer's copies of the English and the Russian editions, so one might at first suppose that this book is more an historical document than a work of present interest. It is pleasant to report that this supposition is false; even though the book includes none of the beautiful and far-reaching results that functional analysis in partially ordered spaces has produced since about 1957, its treatment has many virtues and its contents perennial interest. In this late review, then, it seems appropriate to try to place the work in perspective, to examine briefly what it does and at a little more length what it does not do.

Hewitt's review of the Russian edition (MR **24** #A3494) describes the book as "an updated, abridged, and elementary version of the monograph *Functional analysis in partially ordered spaces* (Russian) by Kantorovič, Vuli(k)h and Pinsker (GITTL, Moscow, 1950; MR **12**, 340)," and gives a chapter-by-chapter list of the contents. One may thus omit the list here, and expand on the description by saying that this is primarily a book about vector lattices, usually complete or σ-complete, which is not really a simple introduction to but a fairly complete account of their theory. The theory, which began its development in the middle thirties, was itself fairly complete by the middle fifties. Two rather old-fashioned characteristics of the theory are particularly notable: its preference for convergence defined in terms of the lattice-order relations to norms and topologies (only two of the thirteen chapters of the present book deal with normed, or even Fréchet, spaces), and its lack of negative surprises (representation theorems show that the examples of vector lattices that come naturally to one's mind are pretty typical, and when one needs a counterexample a fairly familiar object will supply it). So this is an exposition of a nearly finished part of abstract mathematics whose objects look familiar, whose technical armamentarium is modest, and whose results are nearly all complete and satisfying.

One is happy to report that the exposition is a masterful example of mathematics of an elementary character written to be read. A student with a good real variables course as preparation could read the book with virtually no supervision. Definitions are regularly linked to examples. The book steadily pulls the reader along to make him ready for the next idea just before it occurs on the page. Pictures are drawn when pictures are needed. Topics are covered thoroughly as they arise without dissipating the momentum that draws each new concept from its predecessors. If one had to select a part of the book for special mention, it would be the fifth chapter, in which the author introduces the spaces $C_\infty(Q)$ of extended-real-valued continuous functions on an extremally disconnected compact Hausdorff space which take infinite values only on a nowhere-dense set. These spaces are complete vector lattices, and they are "big" enough that they can contain, in a "tightly imbedded" way, any given complete vector lattice with (weak) unit element. Dealing with infinite-valued functions always makes some verifications necessary, and it is possible to work out the properties of the lattices $C_\infty(Q)$ simply by relentless and detailed *ad hoc* checking. Here, however, the author has thought things through very carefully, and the exposition flows just as easily and naturally over these points as it does elsewhere.

By the time one reaches the eleventh chapter of the book, one can observe with pleasure that one knows *in abstracto* enough about strongly closed real algebras of selfadjoint operators on Hilbert space that the standard theorems on their representations as spaces $C(Q)$, the (integral) spectral theorem (even for unbounded operators), and von Neumann's theorem that each such algebra on a separable Hilbert space is the algebra of all bounded Borel functions of a single operator, fall out as corollaries. Indeed, by that time one knows the general theory of vector lattices quite well, and one should be able to read about special topics that the author does not mention—e.g. modulared spaces, or characterizations of the L^p spaces—with ease elsewhere. The part of the book dealing with lattices concludes with a chapter—that does not really fit—on functional equations related to monotonic processes and analogues of Newton's method; the chapter is certainly readable and even enjoyable.

The chapters entitled "Normed lattices and countably-normed lattices" and "Linear functionals," however, begin to indicate what the book does not do, and what an elementary approach costs. One example from each chapter: When the author sets out to characterize (AM)-spaces with unit as spaces $C(Q)$, he is unable to realize the space Q as the set of extremal points (or lattice homomorphisms) in the set of positive linear functionals of norm 1 on the given (AM)-space X [for definitions of these objects as well as the extremal-point realization, see, e.g., H. H. Schaefer, *Topological vector spaces*, Macmillan, New York, 1966, Chapter V, §8]; he must instead resort to forming the completion-by-cuts $\hat{X}$, forming the Stone space of the Boolean algebra that he calls the "base" of the latter (the set of its nonnegative elements e for which $e \perp (1 - e)$), realizing $\hat{X}$ as the lattice of all continuous real-valued functions on that Stone space and then passing to the obvious quotient (identifying those points that X identifies). Again, in order to show that the pointwise limit of a sequence of regular (i.e., order-bounded, or differences-of-positive) linear functionals on a σ-complete vector lattice is a linear functional of the same kind, the author is obliged to give an explicit order-theoretic *bosse glissante* argument—even though the ordinary uniform boundedness principle of functional analysis would make the proposition a triviality and even extend it to pointwise-bounded nets.

Finally, and unfortunately, the chapter on general "Partially ordered spaces" does little more than indicate by omission what has happened in the last fifteen years. The partially ordered normed spaces are those initially considered by M. Kreĭn (indeed, the author explicitly acknowledges his close adherence to (M. Kreĭn and M. Rutman, Uspehi Mat. Nauk **1**, no. 3 (23) (1948), 3–95)); their positive cones possess interior points, so that as far as order is concerned they are ordered-subspaces-containing-the-constants of a space $C(Q)$, Q compact Hausdorff. (The topologies are the same if and only if the cone is normal; but the author deals with the fundamental concept of normal cone only for normed vector lattices with unit!) Because the author is unwilling either to construct the appropriate sublinear functional or to separate the appropriate convex sets in order to apply the standard Hahn-Banach theorem, the section on extension, existence and continuity of positive linear functionals insists that the reader go through the proof of Hahn-Banach all over again, either making changes to take the order structure into account or imposing a new order structure on a space given with another order. The justification for defining normality in the first place, namely that it is equivalent to being able to write every equicontinuous set in the dual as contained in the difference of two positive equicontinuous sets, never appears. The small bow in the direction of operator theory at the end of the chapter—a fixed-point theorem for commutative semigroups of positive operators, with the Markov fixed-point theorem as a corollary thereof—really has nothing to do with order; the material is just as easy to understand in its usual setting, that of an arbitrary convex compact set.

On the one hand, then, the book is a masterful, readable and complete treatment of vector lattices. On the other, it has no insight from functional analysis and convex geometry, little operator theory (no spectral theory, which in view of the remarkable spectral theory of positive operators (Schaefer, op. cit., Appendix; also Acta Math. **107** (1962), 125–173) is particularly regrettable), nothing deep on ordered subspaces of $C(Q)$ (the setting for the profound and profoundly useful theory of representing measures, boundaries, and the simplices of Choquet), not even the Stone-Weierstrass theorem. The reviewer has absolutely no intention to fault the author for his refusal to produce a 1971 treatise in 1961, but the prospective reader should be aware that a contemporary "Introduction to the theory of partially ordered spaces" does not really exist; their publication dates indicate that the book of Schaefer cited above, and (A. L. Peressini, *Ordered topological vector spaces*, Harper and Row, New York, 1967) cannot go far enough.

The typography and even the paper of the book are very fine. The translator should be thanked for rendering the terminology of the 1930's school of Kantorovič into terminology related to the rest of mathematics. Except for passages written in international mathematical jargon, however, his translation has a genius for choosing the wrong synonym equaled only by that remarkable lady in Sheridan's *Rivals*. The reviewer will never be able to understand how a book whose language is native neither to its author, its translator (apparently!) nor its editors can be set in type without having been read for language by a native speaker. Producing mathematics books is an extremely expensive process; why make errors that would be so cheap to avoid?

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