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SCHUR MULTIPLIERS OF THE KNOWN FINITE SIMPLE GROUPS[1]

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ABSTRACT. In this note, we announce some results about the Schur multipliers of the known finite simple groups. Proofs will appear elsewhere. We shall conclude with a summary of current knowledge on the subject.

Basic properties of multipliers and covering groups of finite groups are discussed in [**6**]. Notation for groups of Lie type is standard [**3**], [**8**]. G' denotes the commutator subgroup of the group G, $Z(G)$ the center of G, Z_n the cyclic group of order n; other group theoretic notation is standard (see [**5**] or [**6**]). $M_p(G)$ denotes the p-primary component of the multiplier $M(G)$ of the finite group G. $m(G)$ is the order of $M(G)$ and $m_p(G)$ is that of $M_p(G)$. Also, q denotes a power of the prime p.

We describe these results in a sequence of theorems.

THEOREM 1. $m_2(G_2(4)) = 2$, $m_3(G_2(3)) = 3$, $m_2(F_4(2)) = 2$.

In each of these cases, generators and relations for the (unique) covering group are given.

THEOREM 2. $M(^2A_2(q)) \cong Z(SU(3, q))$, *i.e.* $m(SU(3, q)) = 1$.

THEOREM 3. *Let G be a Steinberg variation defined over a finite field of characteristic p, i.e.* $G = {}^2A_n(q)$, $n \geqq 2$, ${}^2D_n(q)$, $n \geqq 4$, ${}^3D_4(q)$, *or* ${}^2E_6(q)$. *Then* $M_p(G) = 1$ *except for*

$$M_2(^2A_3(2)) \cong Z_2, \qquad M_3(^2A_3(3)) \cong Z_3 \times Z_3,$$

$$M_2(^2A_5(2)) \cong Z_2 \times Z_2, \qquad M_2(^2E_6(2)) \cong Z_2 \times Z_2.$$

THEOREM 4. *If G is a Ree group of type F_4, then* $m(G) = 1$.

THEOREM 5. *The Tits simple group* ${}^2F_4(2)'$ *has trivial multiplier.*

THEOREM 6. *The sporadic groups below have multipliers of the stated orders.*

AMS 1970 *subject classifications.* Primary 20C25, 20D05, 20G40; Secondary 20G05, 20F25.

Key words and phrases. Schur multiplier, covering group, finite group of Lie type, sporadic group, simple group.

[1] This research is from the author's Ph.D. Thesis, University of Chicago, 1971, written under the supervision of Professor John G. Thompson and with the financial support of an N. S. F. Graduate Fellowship.

Held		1
Suzuki		6
Conway's	$\cdot 3$	1
Fischer's	$M(22)$	6
	$M(23)$	1
	$M(24)$ (*nonsimple*)	1
	$M(24)'$	$3^n, n \geqq 0$.

The relevant covering groups for Theorem 1 and for ${}^2E_6(2)$ were constructed by the author. In the other cases, the author obtained an upper bound on the order of the multiplier which was equal to the order of the center of a known perfect group whose central quotient is the simple group under consideration.

The theorems of Steinberg [**9**], [**10**] describe the p'-part of the multipliers of finite Chevalley groups, Suzuki groups, Ree groups of type F_4, and Steinberg variations except type ${}^2A_n$, n even (the odd-dimensional unitary groups). In [**9**] Steinberg shows that, for Chevalley groups, if the cardinality of the field is large enough, the p-part of the multiplier is trivial. The same holds for any field if the rank is large enough. Furthermore, he has determined (unpublished) the list of Chevalley groups with nontrivial (or possibly nontrivial) p-parts to their multiplier. Theorem 1 is the author's contribution to settling three cases on this list. The author has determined a similar list (Theorems 3 and 4) for the Steinberg variations and the Ree groups of type F_4. Recently, Steinberg has informed the author that he has settled the p'-parts for ${}^2A_n$, n even, $n \geqq 4$; they are $Z(SU(n+1, q))$, as expected. Theorem 2 is the author's contribution in this area. Thus, the multipliers of all finite groups of Lie type have been determined.

Using other methods, multipliers of some twisted groups of Lie type have been handled by Alperin and Gorenstein [**1**] (for the Suzuki groups ${}^2B_2(2^{2n+1})$ and the Ree groups ${}^2G_2(3^{2n+1})$) and by Ward [**13**] (for the Ree groups ${}^2F_4(2^{2n+1})$, $n \geqq 2$). The author has settled the latter case for all n, and for ${}^2F_4(2)'$ (Theorems 4 and 5), without appealing to Ward's results.

In the following two tables, we present, to the best of the author's knowledge, the multipliers of the known simple groups. In the first table, only "exceptional" groups of Lie type appear—those for which the characteristic of the defining field divides the order of the multiplier. "Accidental" isomorphisms are noted (which may partly explain the exceptional nature). Results here not due to Steinberg or to those noted above are by Thompson, Burgoyne (independently) for $A_2(4)$ and Fischer, Rudvalis, Steinberg for $B_3(3)$. In the second table, the results are due to Burgoyne, Fong [**2**] for the Mathieu groups, Janko [**7**] for J_1, Wales, McKay [**11**] for J_2 and J_3, Wales, McKay [**12**] (and the author, independently) for the Higman-Sims group, Thompson for McLaughlin's group and Lyons' group. A similar table has appeared in a recent article by Feit [**4**], but our table contains some corrections of the author's former results.

TABLE 1
Exceptional groups of Lie type

Exceptional Chevalley group	*Multiplier*	*Exceptional twisted group of Lie type*	*Multiplier*
$A_1(4) \cong \mathrm{Alt}(5)$	Z_2	${}^2A_3(2) \cong C_2(3)$	Z_2
$\cong A_1(5)$		${}^2A_3(3)$	$Z_3 \times Z_{12}$
$A_1(9) \cong \mathrm{Alt}(6)$	Z_6	${}^2A_5(2)$	$Z_2 \times Z_6$
$A_2(2) \cong A_1(7)$	Z_2	${}^2B_2(8)$	$Z_2 \times Z_2$
$A_2(4)$	$Z_4 \times Z_{12}$	${}^2E_6(2)$	$Z_2 \times Z_6$
$A_3(2) \cong \mathrm{Alt}(8)$	Z_2		
$B_2(2) \cong \mathrm{Sym}(6)$	Z_2		
$B_3(2) \cong C_3(2)$	Z_2		
$B_3(3)$	Z_6		
$D_4(2)$	$Z_2 \times Z_2$		
$F_4(2)$	Z_2		
$G_2(3)$	Z_3		
$G_2(4)$	Z_2		

TABLE 2
Sporadic groups

Sporadic group	*Multiplier*	*Sporadic group*	*Multiplier*
M_{11} (*Mathieu's groups*)	1	*McLaughlin*	Z_3
M_{12}	Z_2	*Suzuki*	Z_6
M_{22}	Z_6	$\cdot 1$ (*Conway's groups*)	$2n, n \geqq 1$
M_{23}	1	$\cdot 2$	?
M_{24}	1	$\cdot 3$	1
J_1 (*Janko's groups*)	1	$M(22)$ (*Fischer's groups*)	Z_6
J_2	Z_2	$M(23)$	1
J_3	Z_3	$M(24)'$	$3^n, n \geqq 0$
Held	1	$M(24)$ (*nonsimple*)	1
Higman-Sims	Z_2	*Lyons*	1

REFERENCES

1. J. L. Alperin and D. Gorenstein, *The multiplicators of certain simple groups*, Proc. Amer. Math. Soc. **17** (1966), 515–519. MR **33** #1362.

2. N. Burgoyne and P. Fong, *The Schur multipliers of the Mathieu groups*, Nagoya Math. J. **27** (1966), 733–745; Correction, ibid. **31** (1968), 297–304. MR **33** #5707; **36** #2705.

3. R. W. Carter, *Simple groups and simple Lie algebras*, J. London Math. Soc. **40** (1965), 193–240. MR **30** #4855.

4. W. Feit, *The current situation in the theory of finite simple groups*, Proc. of 1970 International Congress of Mathematicians (to appear).
5. D. Gorenstein, *Finite groups*, Harper and Row, New York, 1968. MR **38** #229.
6. B. Huppert, *Endliche Gruppen.* I, Die Grundlehren der math. Wissenschaften, Band 134, Springer-Verlag, Berlin, 1967. MR **37** #302.
7. Z. Janko, *A new finite simple group with abelian Sylow* 2-*subgroups and its characterization*, J. Algebra **3** (1966), 147–186. MR **33** #1359.
8. R. Steinberg, *Lectures on Chevalley groups*, Yale University Notes, New Haven, Conn., 1967.
9. ———, *Générateurs, relations, et revêtements de groupes algébriques*, Colloq. Théorie des Groupes Algébriques (Bruxelles, 1962), Librairie Universitaire, Louvain, Gauthier-Villars, Paris, 1962, pp. 113–127. MR **27** #3638.
10. ———, *Representations of algebraic groups*, Nagoya Math. J. **22** (1963), 33–56. MR **27** #5870.
11. D. Wales and J. McKay, *The multipliers of the simple groups of order* 604,800 *and* 50,232,960, J. Algebra **17** (1971), 262–272.
12. ———, *The multiplier of the Higman-Sims group* (to appear).

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