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ON CONJUGATE POWERS IN EIGHTH-GROUPS

BY SEYMOUR LIPSCHUTZ

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Let G be a finitely presented group with generating elements $a_1, \cdots, a_\lambda$ and defining relations $R_1 = 1, \cdots, R_\mu = 1$. We assume without loss of generality that the *relators* R_i form a symmetric set, i.e. that the R_i are cyclically reduced and are closed under the operations of taking inverses and cyclic transforms. We call G an *eighth-group* if it satisfies the following condition:

(*) If $R_i \cong XY$ and $R_j \cong XZ$ are distinct relators, then the length of the common initial segment X is less than 1/8 the length of either relator.

A classical example of such a group is the fundamental group G_k of an orientable closed 2-manifold of genus $k > 2$; it has the presentation

$$G_k = \text{gp}(a_1, b_1, \cdots, a_k, b_k; a_1 b_1 a_1^{-1} b_1^{-1} \cdots a_k b_k a_k^{-1} b_k^{-1} = 1).$$

More generally, the Fuchsian groups $F(p; n_1, \cdots, n_d; m)$, see Greenberg [3], are eighth-groups if $4p + d + m, n_1, \cdots, n_d > 8$.

The class of eighth-groups were first considered by Greendlinger who solved the word problem [4] and the conjugacy problem [5] for them. Similar "small cancellation" groups have been studied by Tartakovskii [8], Britton [1], Lyndon [6] and Schupp [7], among others.

We now state our main result.

THEOREM. *Suppose W is an element of infinite order in an eighth-group G. If $|m| \neq |n|$ then W^m and W^n are in different conjugacy classes. In particular, $W, W^2, W^3, \cdots$ are in different conjugacy classes.*

This theorem has already been known to hold for the above fundamental groups G_k and for the Fuchsian groups; but all the proofs have been topological. Our theorem holds for a much wider class of groups and, moreover, the proof is purely algebraic.

The author conjectures that the theorem also holds for the small concellation groups in general.

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We also note that Gowdy [2] has classified those elements in eighth-groups which are conjugate to their inverses, and showed that no such elements exist if the group is torsion-free.

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Temple University, Philadelphia, Pennsylvania 19122