FREE EXTERIOR POWERS

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Our purpose is to announce several results concerning a unital module M over a commutative ring R with unity.

THEOREM 1. *Let M be a module over the ring R. Suppose for some p that $\bigwedge^p M$ is free of rank one. Then*

(i) *M is finitely generated, projective, and reflexive;*

(ii) *the module $M^* = \mathrm{Hom}(M, R)$ has the same properties, and $\bigwedge^p M^*$ is free of rank 1; and*

(iii) *for each q, $0 \leqq q \leqq p$,*

$$\bigwedge^q M^* \approx (\bigwedge^q M)^* \approx \bigwedge^{p-q} M.$$

COROLLARY 1. *If the module M of Theorem 1 has a pure p-vector for a basis, then M is free.*

COROLLARY 2 (H. OSBORN). *If the ring R in Theorem 1 is a local ring, then M is free.*

THEOREM 2. *Let M be a module over the ring R. Suppose for some p that $\bigwedge^p M$ is cyclic. Then $\bigwedge^{p+1} M = 0$.*

THEOREM 3. *Let M be a module over the ring R. Suppose for some p that $\bigwedge^p M$ is free of finite rank q. Then $\bigwedge^{p+q} M = 0$.*

A detailed paper will be submitted elsewhere.

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