A NOTE ON SUPPORT POINTS

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Communicated by Ernest Michael, May 27, 1968

Bishop and Phelps proved in [1] that every proper closed convex subset of a Banach space has "many" support points. (A *support point* of a convex set C in a real topological linear space E is a point x of C for which there is a nonzero continuous linear functional f on E with $f(x) = \sup_{c \in C} f(c)$.) At the end of their paper they asked whether "Banach space" can be replaced by "complete locally convex space" in the statement of this result. In [3], Klee settled this negatively by exhibiting a proper closed convex set in $R^{\aleph_0}$ with no support points. The set in Klee's example is unbounded, and at the end of [3] the question was raised whether a bounded closed convex subset of a complete locally convex space must have support points. (Note that a bounded closed convex subset of $R^{\aleph_0}$ is weakly compact and hence has support points. Indeed, if $\{B_i\}_{i=1}^{\infty}$ is any sequence of reflexive Banach spaces, every bounded closed convex subset of the product space $\prod_{i=1}^{\infty} B_i$ is weakly compact and therefore has support points.)

We settle Klee's question negatively:

THEOREM. *Let $\{B_i\}_{i=1}^{\infty}$ be any sequence of nonreflexive Banach spaces. Then in the product space $\prod_{i=1}^{\infty} B_i$ there is a closed bounded convex set which has no support points.*

The construction of the example rests on the theorem of James [2], that on every nonreflexive Banach space B, there is a continuous linear functional which does not assume its supremum on the unit ball of B. The detailed proof and related results will appear elsewhere.

REFERENCES

1. Errett Bishop and R. R. Phelps, *The support functionals of a convex set*, Proc. Sympos. Pure Math., vol. 7, Amer. Math. Soc., Providence, R. I., 1963, pp. 27–35.

2. R. C. James, *Weakly compact sets*, Trans. Amer. Math. Soc. **13** (1964), 129–140.

3. V. L. Klee, *On a question of Bishop and Phelps*, Amer. J. Math. **85** (1963), 95–98.

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