REFERENCES

1. S. Bochner, *Boundary values of analytic functions in several variables and of almost periodic functions*, Ann. of Math. **45** (1944), 708–722.
2. K. de Leeuw, and I. Glicksberg, *Quasi-invariance and analyticity of measures on compact groups*, Acta Math. **109** (1963), 179–205.
3. H. Helson, *Compact groups with ordered duals*, Proc. London Math. Soc. **XIV** A (1965), 144–156.
4. ———, *Invariant subspaces*, Academic Press, New York, 1964.
5. H. Helson and D. Lowdenslager, *Prediction theory and Fourier series in several variables*. I, Acta Math. **99** (1958), 165–201.
6. ———, *Prediction theory and Fourier series in several variables*. II, Acta Math. **106** (1961), 175–212.
7. G. Kallianpur and V. Mandrekar, *Representation and multiplicity of purely non-deterministic stochastic processes*, Theory of Probability and its Applications **X** (1966).
8. G. W. Mackey, *A theorem of Stone and von Neumann*, Duke Math. J. **16** (1949), 313–326.
9. M. G. Nadkarni, *On a class of measures on Bohr group*, Pacific J. Math. (to appear).
10. J. von Neumann, *Die Eindeutigkeit der Schrödingerschen Operatoren*, Math. Ann. (1931), 570–578.
11. F. Riesz and B. Sz-Nagy, *Functional analysis*, Ungar, New York, 1955.
12. W. Rudin, *Fourier analysis on groups*, Interscience, New York, 1962.

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A NONLINEAR BOUNDARY VALUE PROBLEM

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1. Introduction. The main result of this paper establishes the existence of solutions of certain nonlinear two point boundary value problems for a class of nonlinear second order differential equations.

A corollary to the main theorem includes a boundary value problem recently considered by Herbert B. Keller [1] and Klaus Schmitt [2].

2. Definitions. In the following definitions let **S** stand for a point set in the YZ-plane.

$A = \{S: S \text{ is an arc}\}$,
$H_1 = \{S: (Y_1, Z_1), (Y_2, Z_2) \in S \Rightarrow (Y_1 - Y_2)(Z_1 - Z_2) \geqq 0\}$,
$H_2 = \{S: (Y_1, Z_1), (Y_2, Z_2) \in S \Rightarrow (Y_1 - Y_2)(Z_1 - Z_2) \leqq 0\}$,
$J_1 = \{S: \forall \exists\ (Y, Z) \in S \ni Z = N\}$,
$J_2 = \{S: \forall \exists\ (Y, Z) \in S \ni Y - Z = N\}$,
$R = \{(X, Y, Z): X_1 \leqq X \leqq X_2, \ |Y| + |Z| < \infty\}$,
$B_0 = \{f(X, Y, Z): f \text{ is continuous in } R\}$,

$B_1 = \{f(X, Y, Z): Y_1 > Y_2 \Rightarrow f(X, Y_1, Z) > f(X, Y_2, Z)\}$,
$B_2 = \{f(X, Y, Z): \exists \text{ constant } K \ni |f(X, Y, Z_1) - f(X, Y, Z_2)| \leq K|Z_1 - Z_2|\}$.

3. The main theorem. *Let L_1 and M_1 be in $A \cap H_1 \cap J_1$ and let L_2 and M_2 be in $A \cap H_2 \cap J_2$. Let M_1 be bounded above by L_1; let M_2 be bounded above and to the right by L_2, in the sense that there are no points $(Y_M, Z_M) \in M_2$ and $(Y_L, Z_L) \in L_2$ such that $Y_M > Y_L$ and $Z_M > Z_L$. Let P_1 be a connected set in the region of the YZ-plane bounded by L_1 and M_1, and let P_2 be a connected set in the region of the YZ-plane bounded by L_2 and M_2. Let $P_1 \in J_1$, let $P_2 \in J_2$ and let one of the sets P_1 and P_2 be closed.*

THEOREM. *If $F_a(X, Y, Z)$, $F_b(X, Y, Z)$, and $f(X, Y, Z)$ are in B_0, F_a and F_b are in $B_1 \cap B_2$, and $F_a(X, Y, Z) > f(X, Y, Z) > F_b(X, Y, Z)$ for all $(X, Y, Z) \in R$, then there is a $y(X) \in C^2[X_1, X_2]$ such that $y''(X) = f(X, y(X), y'(X))$ for all $X \in [X_1, X_2]$, $(y(X_1), y'(X_1)) \in P_1$ and $(y(X_2), y'(X_2)) \in P_2$.*

The proof, which will be given in detail elsewhere, utilizes properties of solution funnels of continuous differential equations, developed by H. Kneser [3] and M. Fukuhara [4], and existence theorems for a more restricted class of boundary value problems by M. Lees [5] and J. W. Bebernes [6].

The significance of the theorem is as follows: the function $f(X, Y, Z)$ in the differential equation need not be locally smooth in Z (i.e., no Lipschitz condition is imposed), nor need $f(X, Y, Z)$ be nondecreasing in Y; the nonlinear boundary sets P_1 and P_2 are quite general, and in particular need not be differentiable curves.

REFERENCES

1. H. B. Keller, *Existence theory for two point boundary value problems*, Bull. Amer. Math. Soc. **72** (1966), 728–731.

2. Klaus Schmitt, *Solutions to boundary value problems and periodic solutions of second-order nonlinear differential equations*, Ph.D. Thesis, University of Nebraska, Lincoln, Nebr., 1967.

3. H. Kneser, *Ueber die Lösungen eines Systems gewöhnlicher Differentialgleichungen das der Lipschitzschen Bedingung nicht genügt*, S.-B. Preuss. Akad. Wiss. Phys.—Math. Kl. II (1923), 171–174.

4. M. Fukuhara, *Sur l'ensemble des courbes integrales d'un systeme d'équations différentielles ordinaires*, Proc. Imperial Acad. Japan **4** (1928), 448–449.

5. M. Lees, *A boundary value problem for nonlinear ordinary differential equations*, J. Math. Mech. **10** (1961), 423–430.

6. J. W. Bebernes, *A subfunction approach to a boundary value problem for ordinary differential equations*, Pacific J. Math. **13** (1963), 1053–1066.

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