BOOK REVIEW

Mathematical foundations of quantum mechanics, Mathematical Physics Monograph Series, By G. W. Mackey, W. A. Benjamin, New York, 1963. x+137 pp.

This book is a revised version of lecture notes written by the author for a course at Harvard. The aim is stated in the preface: "to explain quantum mechanics and certain parts of classical physics from a point of view more congenial to pure mathematicians than that commonly encountered in physics texts. Accordingly, the emphasis is on generality and careful formulation rather than on the technique of solving problems. On the other hand, no attempt is made at complete rigor. In places a complete treatment would have taken us too far afield and in others nontrivial mathematical problems remain to be solved. There are also places where completeness simply seemed more troublesome than illuminating. In sum, we have tried to present an outline of a completely rigorous treatment which can be filled in by any competent mathematician modulo the solution of certain more or less well-defined mathematical problems."

Several virtues of the book are

1. An emphasis on coordinate-independent formulations.
2. Insistence on clear statement of assumptions at all stages of the game, making it clear which are based on experiment, which are a priori, and which are technical mathematical assumptions.
3. A clear distinction between analogy and isomorphism.
4. The excellent little historical discussions which appear at various points throughout.

Chapter 1 begins with an outline of Newtonian mechanics, continues with an exposition of C_∞ manifolds, and then treats Hamiltonian systems in this context. There is a discussion of linear systems, in which an intrinsic complex Hilbert space structure can be introduced so that the Hamiltonian is half of the square of the norm and the dynamical group is a one-parameter unitary group. This is then (sketchily) extended to infinite-dimensional linear systems. Finally, there is a brief treatment of statistical mechanics and thermodynamics, including an information-theoretic "justification" (discovered independently by Mackey) of the use of the Gibbs ensemble.

Chapter 2, on quantum theory, begins with a bit of history. Then there is given a detailed account of some of the author's ideas on the logical and statistical basis of quantum theory. This is perhaps the most idiosyncratic part of the book; the ideas developed here have subsequently been pushed further by various authors such as Gleason, Pisot, Varadarajaran. The abstract Schrödinger equation is derived from this discussion. Canonical quantization rules are given (in a coordinate-free manner) for the finite-dimensional Hamiltonian systems of the first section. For linear systems, the alternate "Fock-Cook" method of quantization is described, which, of course, applies to infinite-dimensional systems as well. Finally, there is a brief discussion of quantum statistics.

The third (and last) chapter begins by giving some background on group representation theory, preliminary to an extremely lucid account of the use of group theory and quantum mechanics in explaining the periodic table.

While the author apologizes in his Preface for the possible errors which might have arisen in publishing a set of lecture notes, this reviewer found the book remarkably error free by the usual standards of mathematical publication. However, the style and notation tend to be a little casual. Also, formulae involving superscripts and subscripts are frequently hard to read because of closeness of lines of type.

All in all, the book is an interesting, lucid, and original treatment of the foundations of quantum physics, and one which mathematicians in particular should find a useful complement to the usual physics texts.

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