Hence the C^*-algebra $\pi_{\chi}(A)$ and so A have a type III-factor $*$-representation.

This completes the proof.

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UNIVERSITY OF PENNSYLVANIA

SOME UNSYMMETRIC COMBINATORIAL NUMBERS

BY ANDREW SOBCZYK

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By an *n-configuration* we shall mean an abstract set of n elements, together with the set of all unordered pairs of distinct elements from the set. It is convenient also to use quasi-geometrical terminology such as *vertex* for element, *edge* or *side* for a pair (2-tuple), *triangle* as well as triple (3-tuple) for a 3-subconfiguration, and so on.

The Ramsey number $N(p, q, 2)$ (see [**3**, pp. 38–43], or [**2**, pp. 61–65]), for two kinds h, v of pairs (or two "colors of edges"), is the smallest integer such that if $n \geqq N(p, q, 2)$, then any n-configuration is sure to contain *either* an h p-tuple (a p-tuple all of whose edges are h) *or* a v q-tuple. Call a p-tuple all of whose edges are alike (h or v) a *like* p-tuple. We introduce, and partially determine the values of, new analogous combinatorial numbers $K(p, q, 2)$, $M(p, q, 2)$, and $V(p, q, 2)$.

DEFINITIONS. The number $K(p, q, 2)$ is the smallest integer such that if $n \geqq K(p, q, 2)$, then for each vertex, the configuration is sure to contain *either* a like p-tuple containing the vertex, *or* a like q-tuple not containing the vertex. For three kinds r, g, v of edges, $M(p, q, 2)$

is the smallest integer such that if $n \geqq M(p, q, 2)$, the configuration is sure to contain either a like p-tuple, or a j, k q-tuple (a q-tuple having at most two kinds j, k of edges, where $j, k = r, g$, or v). The number $V(p, q, 2)$ is the smallest integer such that if $n \geqq V(p, q, 2)$, then *for each vertex* of the configuration, the configuration contains either a like p-tuple containing the vertex, or a j, k q-tuple not containing the vertex.

Consider for a moment "verticial" numbers, which otherwise are like the Ramsey numbers: $S(p, q, 2)$, for example, is the smallest integer such that a configuration with $n \geqq S(p, q, 2)$ is sure to contain, for each vertex, either an h p-tuple containing the vertex, or a v q-tuple not containing the vertex. Evidently $N(p, q, 2) \leqq S(p, q, 2)$. But for all $p \geqq 3$, $q \geqq 3$, $S(p, q, 2) = \infty$: for arbitrarily large n, at one vertex, assign $(p-1)$ edges from the vertex to be h, the remainder v. Let one edge joining a pair of other ends of the $(p-1)$ edges be v, and let all other edges of the n-configuration be h. Then for the vertex, the n-configuration contains neither an h p-tuple containing the vertex, nor a v q-tuple not containing the vertex. Moreover $S(p, q, 2) = \infty$ for all $p \geqq 2$, $q \geqq 2$.

Denote by $W(q, p, 2)$ the smallest integer such that if $n \geqq W(q, p, 2)$, then for each vertex the configuration is sure to contain either a j,k q-tuple containing the vertex, or a like p-tuple not containing the vertex. We notice that $W(q, p, 2) = V(p, q, 2)$.

Our results so far concerning the numbers K, M, V are indicated in the following Theorem 1 (including the table) and Theorem 2. For purposes of comparison, the known values of the Ramsey numbers N also are included: the entries in the table are the values of N, K, M, V in that order, for each p, q.

THEOREM 1. *For p, q from 3 to 5 inclusive, the numbers have values as given in the following table (cf. the table in* [**3**, p. 42]).

$p \backslash q$	3	4	5
3	6, 6, 5, 6	9, 8, 8, 10 or 11	14, 10, 14, 14
4	9, 7, 5, 6	18, 18, 10 to 17, 11 to 18	
5	14, 7, 5, 6		

For all $p > 3$, we have that $K(p, 3, 2) = 7$; for all $q \geqq 3$, that $K(q, q, 2) = N(q, q, 2)$ and $K(3, q, 2) = 2q$; for $p > q$, that $K(p, q, 2) \leqq N(q, q, 2) + 1$; and for $q > p > 3$, that $K(p, q, 2) \geqq \max(K(p-1, q, 2) + 1, 2q + p - 3)$. Further, $N(3, 6, 2)$ is 17 or 18 (cf. [**1**]), *and $17 = M(3, 6, 2) \leqq N(3, 6, 2)$.*

A configuration is called *degenerate, with respect to* any of the combinatorial numbers, in case it does contain (for each vertex in case of V) either a p-tuple or a q-tuple as described in the corresponding definition. As an example, it is quite easy to find an 8-configuration which is nondegenerate with respect to $N(3, 4, 2)$—an octagon with an 8-cycle plus a 4-cross (see below) of blue edges, and an 8-cycle plus two 4-cycles of red edges, has neither a blue triangle nor a red quadruple (cf. the existence proof for the 8-configuration in [1]). The method of establishing the above lower bounds L is to exhibit in each case a nondegenerate configuration with $n=L-1$. To establish an upper bound U, it is sufficient to show that any configuration with $n=U$ must be degenerate. The value of a combinatorial number of course is determined in case $L=U$. Details will be included in a paper which will be offered for publication elsewhere.

A subsidiary result, analogous to Steiner triple systems ([2] or [3]), is the following. A *k-cycle* is a closed string of k successively adjacent edges, such as $12; 23; \cdots; k-1, k; k, 1$; where $\{1, \cdots, k\}$ is a subset of k of the vertices of the configuration. In any $(2n+1)$-configuration, the edges can be covered (each exactly once) by n k-cycles with $k=(2n+1)$. A *k-cross* is a set of k edges, no two of which are adjacent. The edges of any $(2n+2)$-configuration can be covered by n $(2n+2)$-cycles and an $(n+1)$-cross.

THEOREM 2. *We have* $K(p, q, 2) \leqq N(p, q, 2)$, $M(p, q, 2) \leqq N(p, q, 2)$, $M(p, q, 2) \leqq V(p, q, 2)$. *For each* q, $V(q, q, 2)$ *is either* $M(q, q, 2)$ *or* $M(q, q, 2)+1$; *for any* p, $V(p, q, 2) \leqq M(q, q, 2)+1$. *For* $p>q$, $M(q, q, 2) \leqq \min(M(p, q, 2), V(p, q, 2))$, *and* $V(q, q, 2) \leqq V(p, q, 2)$. *For* $p \geqq 3$, $M(p, 3, 2)=5$, *and* $V(p, 3, 2)=6$. *For* $q>3$, $V(3, q, 2) \leqq (3q-1)$.

Reference [4], for example, indicates the wealth of possible applications for combinatorial results.

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CLEMSON UNIVERSITY