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PURE SUBGROUPS HAVING PRESCRIBED SOCLES

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Let $B=\sum B_n$ be a direct sum of cyclic groups where, for each positive integer n, $B_n=\sum C(p^n)$ is zero or homogeneous of degree p^n where p is a fixed prime. Denote by $\bar{B}$ the torsion completion of B in the p-adic topology. Following established terminology [**1**], we refer to $\bar{B}$ as the closed primary groups with basic subgroup B. A primary group G is said to be pure-complete if each subsocle of G supports a pure subgroup of G. A semi-complete group was defined by Kolettis in [**6**] to be a primary group which is the direct sum of a closed group and a direct sum of cyclic groups.

For a particular B, I exhibited in [**3**] nonisomorphic pure subgroups H and K of $\bar{B}$ having the same socle. Using this example, Megibben [**7**] was the first to show the existence of a primary group without elements of infinite height which is not pure-complete. We mention that each semi-complete group is pure-complete [**4**]. The purpose of this note is to announce the following theorem and corollaries; proofs will appear in another paper.

THEOREM. *Suppose that B is unbounded and countable and that S is any proper dense subsocle of $\bar{B}$ such that $|S|=2^{\aleph_0}$. Then S supports more than $2^{\aleph_0}$ pure subgroups of $\bar{B}$ which are isomorphically distinct.*

The theorem has the following implications.

COROLLARY 1. *Suppose that B is unbounded and countable and that*

G is an uncountable, semi-complete, pure subgroup of $\bar{B}$ such that $B \subseteq G \subset \bar{B}$. Then there is a pure subgroup H of $\bar{B}$ such that $H[p] = G[p]$ and such that H is not pure-complete.

The next result shows that a conjecture stated in [8] is false.

COROLLARY 2. *The direct sum of two pure-complete groups need not be pure-complete.*

A reduced primary group G is said to be quasi-closed if the closure in the p-adic topology of each pure subgroup of G is again a pure subgroup of G. It was shown in [5] that the class of quasi-closed groups is more general than the class of closed groups. This settled a question raised by Head in [2]. It is well known that a closed group is uniquely determined by its Ulm invariants [6]; however, we now have

COROLLARY 3. *There exist more than $2^{\aleph_0}$ quasi-closed groups having the same Ulm invariants.*

Finally, we remark that our theorem is further evidence that the problem of classifying the pure subgroups between B and $\bar{B}$ is difficult (see [1]).

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