BOOK REVIEWS

PCT, *Spin and statistics, and all that.* By R. F. Streater and A. S. Wightman. Benjamin, New York, 1964. (Mathematical Physics Monograph Series, Vol. 1) viii+181 pp.

The most important problem of present-day physics is without doubt that of understanding the interactions between "elementary" particles. The first proposed solution to this problem has been Quantum Field Theory.

Field theory has permitted the calculation with extraordinary accuracy of the phenomena involving electromagnetic interactions (Quantum Electrodynamics) and was the main source of the "dispersion relations" which are so fundamental in the study of strong interactions. Field theory has, however, been plagued from its beginnings with extremely serious mathematical inconsistencies and has given (until now at least) only partial insight into the problem of strong interactions. These facts have led part of the physicists to believe that the notion of field is inadequate for the understanding of the interaction of particles. Even before that, however, various other physicists started to investigate systematically the possibility of putting field theory on a firm mathematical basis, creating Axiomatic Field Theory (AFT).

In AFT one postulates the existence of fields (as "operator-valued distributions") satisfying a certain number of requirements (the Wightman axioms) of physical origin. The "main problem of quantum field theory," as it is called by Wightman and Streater, remains to see if there exist nontrivial fields satisfying these axioms. It is indeed known that one can describe noninteracting particles by "free" fields, but it is not known if interacting particles can be represented by fields satisfying the Wightman axioms. As of now, the main results of AFT are therefore not on the solution of the "main problem" but on the possibility of obtaining physically relevant consequences of the axioms in a mathematically rigorous manner. Two books are now available describing these results, that under review and one by R. Jost (Lectures in Applied Mathematics, Vol. IV, Amer. Math. Soc.) which the reviewer has not yet seen.

Wightman and Streater begin their book by a short review of relativistic quantum mechanics, mostly describing Wigner's work on the subject.

The second chapter of the book, which is also the longest, is on "some mathematical tools." First a brief introduction to the theory of distributions and functions of several complex variables is given. The Laplace transformation of distributions is then studied in detail. Finally, more specific tools are introduced. One is a theorem (Bargmann, Hall, and Wightman) asserting that if a function is analytic in a certain tube (the "forward" tube) of C^{4n} and transforms covariantly under the Lorentz group, it is the restriction of a function analytic in a larger domain of C^{4n} (the "extended" tube) and covariant under the complex Lorentz group. Another specific tool is the "edge of the wedge" theorem. This is a nontrivial generalization to several variables of Painlevé's theorem on couples of analytic functions having the same boundary values on the real axis.

In these first two chapters only the notions and results to be used later are introduced (this, by the way, leads to a somewhat unusual notion of "distributions with fast decrease" on p. 40). These two chapters should, however, make the last two chapters accessible to a physicist without special mathematical background.

The third chapter presents and discusses the axioms of a field theory and gives a proof of Wightman's theorem that a field theory can be reconstructed from the vacuum expectation values of the fields, the various properties of a field corresponding to various properties of the vacuum expectation values.

Physically relevant results of AFT are presented in the fourth chapter. Several developments which turned out not to have the anticipated physical interest have been rightly omitted. Some recent developments have also been omitted, in particular, collision theory, and this may be considered as a good idea in such a pedagogical book since it is now clear that the evolution of the subject is not completed. What remains are the "classical" results: the PCT theorem (Jost), the theorem of connection between spin and statistics, and Haag's theorem. The notion of Borchers classes is also introduced and the Reeh-Schlieder and irreducibility theorems as well as the global nature of local commutativity are proved.

With this book the authors have succeeded in the difficult task of giving a neat, homogeneous presentation of the "classical" part of AFT which until now was buried in the literature and really understood only by a relatively small "club" of specialists. A number of clarifying remarks are made: on the physical significance of analyticity (p. 43), on symmetries (§§3–5), and many others. This is an excellent book, written with unusual competence and vigour, a book that should be read by every theoretical physicist or mathematician interested in physics. It will, however, probably have fulfilled its main aim if it can convince young physicists that, despite a widespread snobbism to the contrary, correct mathematics is a proper tool for obtaining physically relevant results.

D. RUELLE

Probabilities on algebraic structures. By Ulf Grenander. Wiley, New York, 1963. 218 pp. $12.00.

The classical limit theorems of probability theory exemplify what may be termed the "large-number phenomenon." Stated roughly it is this: in combining a large number of independent random variables subject to certain "mild" restrictions, the outcome will be asymptotically either a well-determined number, or a random variable with a well-determined distribution. For example, in one version of the central limit theorem, we form $f(x_1, \cdots, x_n) = n^{-1/2}(x_1 + \cdots + x_n)$, where the x_i are independent random variables with distribution functions F_i subject only to the restrictions:

$$\int |x|^3 dF_i(x) < M < \infty, \quad \int x^2 dF_i(x) = \sigma^2, \quad \int x\, dF_i(x) = 0.$$

The conclusion is that $f(x_1, \cdots, x_n)$ is asymptotically a normal random variable with mean 0 and variance σ^2.

The variables $x_1, \cdots, x_n$ need not be combined linearly. The following instance of the large-number phenomenon, which illustrates this, has recently received attention for its possible application to nuclear physics. Suppose $x_{11}, \cdots, x_{nn}$ are n^2 random entries in a large symmetric matrix with eigenvalues $\lambda_1, \cdots, \lambda_n$. Let

$$f_u(x_{11}, \cdots, x_{nn})$$

denote the proportions of the eigenvalues which do not exceed $u \max_{1 \leq i \leq n} \lambda_i$. Imposing mild restrictions on the variables $x_{11}, \cdots, x_{nn}$, we find that the functions f_u, which are highly complex functions of the variables, tend to well-determined values as $n \to \infty$. (See Chapter 7 of the book under review for more details.)

Clearly, the elucidation of the scope of this phenomenon is a major problem for probabilists. One of the motivations for studying "probabilities on algebraic structures" is that it provides a systematic approach to this problem. Namely, the instances of the large-number phenomenon that classical probability theory discovered involved addition of real-valued random variables. It is reasonable to expect that by copying its methods one can extend these results to random variables taking values in more general algebraic structures and being combined in accordance with the relevant laws of composition.