

Some background material is collected in six appendices. The book concludes with a note on recent and current research.

Although there are of course many points of contact with Zygmund's famous treatise, Kahane and Salem have chosen their topics so that the actual overlapping is very small. The wealth of interesting material which they present gives convincing proof of the continued vitality of this old and important branch of classical analysis.

WALTER RUDIN

Introduction to the theory of integration. By T. H. Hildebrandt. Academic Press, New York, N. Y., 1963. ix+385 pp. \$14.00.

This book has grown out of the author's courses at the University of Michigan, given over a period of many years, and may be taken as his mature view of the theory of integration. The prerequisites stated in the preface are "a basic knowledge of the topological properties of the real line, continuous functions, functions of bounded variation, derivatives, and Riemann integrals." Actually only elementary properties of the real line and of continuous functions are assumed: everything else is examined in great detail. The point of view is severely classical for the most part, contemporary ideas of integration on locally compact spaces, for example, not being mentioned at all. The author's aim is exactly this, to be sure. He wants to give a concrete treatment which will illumine the procedures of abstract integration theory.

The chapter titles are as follows. I. A general theory of limits. This is standard introductory material. II. Riemannian types of integration. This chapter contains all that anyone could want to know about Riemann-Stieltjes integrals on the line. III. Integrals of Riemann type of functions of intervals in two or higher dimension. This chapter is complicated, and to the reviewer's mind is an excellent argument by itself for the abstract approach to measure theory: functions of bounded variation in several variables are just too unwieldy. IV. Sets. This is a matter of definitions, notation and standard simple facts. V. Content and measure. Here the author treats Jordan content, Lebesgue measure, and Lebesgue-Stieltjes measures on the line. VI. Measurable functions. This chapter contains standard facts, including Luzin's theorem. VII. Lebesgue-Stieltjes integration. VIII. Classes of measurable and integrable functions. This is a treatment of L_p ($0 < p \leq \infty$), including the Riesz-Fischer theorem. IX. Other methods of defining the class of Lebesgue integrable functions. Abstract integrals. This is a short sketch of Daniell's construction of the integral. X. Product measures. Iterated integrals. Fubini theorem. The

treatment given here for product measures in R^2 is based on 2-dimensional interval functions, and seems to the reviewer to be distinctly more complicated than the usual treatment. XI. Derivatives and integrals. Vitali's covering theorem, differentiability of functions of bounded variation, etc., are treated.

No one can quarrel with Professor Hildebrandt's intentions. Analysis should deal with concrete analytic entities, and one can easily lose sight of this in the morass of essential superior integrals, locally negligible sets, etc., of the most abstract treatments of integration theory. It is also true that many U.S. mathematicians [not merely graduate students] are unable to carry out manipulations that any competent Oxford undergraduate could do with ease. But the many avatars of Riemann-Stieltjes integration provide no remedy for this intellectual disease. The Daniell approach to integration, based upon Riemann-Stieltjes integrals for *continuous* functions, carries one quickly and easily to Lebesgue-Stieltjes integrals, and thence to all of the measure theory one can want, in any number of dimensions. Armed with this formidable weapon, one can then attack as many concrete problems as one wishes—everything from pointwise summability of Fourier series in many variables to partial differential equations.

Professor Hildebrandt's book displays the author's mastery of the field, which he has known and contributed to nearly from its beginning. In the reviewer's opinion, however, his book represents the history, and not the future, of integration theory.

EDWIN HEWITT

Diophantine geometry. By Serge Lang. Interscience Tracts in Pure and Applied Mathematics, No. 11. John Wiley and Sons, Inc., New York, 1962. 1+170 pp. \$7.45.

Let $(x) = (x_1, x_2, \dots, x_n)$ be n variables in a given ring R . Let $f(x) = f(x_1, x_2, \dots, x_n)$ be a polynomial with coefficients in a given field F . We suppose to begin with that R and F are sets of algebraic numbers. The term "Diophantine Analysis" may be applied to the two following topics:

(1) Diophantine Equations, and here there is the question of discussing the solution in R of $f(x) = 0$.

(2) Diophantine Approximation, and now the problem is to investigate the lower bound of $|f(x)|$ for all (x) in R .

This book is concerned with probably the three most important developments in the period 1909–1963. There is to begin with the so-called Mordell-Weil Theorem. In 1922, the reviewer showed that if