

which are somewhat more sophisticated than its place in the book would suggest and which may be omitted on first reading. Some material is set in small type to indicate that it is not central to the logical development of the subject. Each chapter ends with a collection of exercises, ranging from those which are easy or routine to those which are more difficult and interesting.

All mathematicians and students of mathematics who are interested in algebraic topology owe the authors a debt of gratitude for the production of this fine text.

W. S. MASSEY

*Opere matematiche. Memorie e Note.* By Vito Volterra. Vol. 1, xxxiii + 604 pp., 1954; vol. 2, 626 pp., 1956; vol. 3, 612 pp., 1957; vol. 4, 540 pp., 1960; vol. 5, 538 pp., 1962. Edited by the Accademia Nazionale dei Lincei, Rome. 40,000 Italian lire.

The Accademia Nazionale dei Lincei, in collaboration with the Consiglio Nazionale delle Ricerche, has successfully completed the enormous task of editing the mathematical papers of Vito Volterra (1860–1940) in the relatively short period of eight years from 1954 to 1962. The committee, always headed by the president of the Accademia, consisted originally of U. Amaldi, L. Amoroso, G. Armellini, U. D'Ancona, E. Freda, J. Pérès, E. Persico, M. Picone, A. Signorini, C. Somigliana and E. Volterra. For later volumes, B. Finzi and B. Segre very successfully carried the burden of the work left by the passing of Amaldi, Armellini, and Somigliana.

The mathematical community owes a debt of gratitude to the committee. It is a sad duty of the reviewer to mention here that after the completion of their work two more members, J. Pérès and A. Signorini, passed away.

The five volumes of these works contain most of Volterra's mathematical papers, notes and memoirs. His well known books, some of which were written in collaboration with Pérès, Hostinski, and others, are of course not included in the present collection. Also omitted are some general addresses, obituaries, reviews, etc.

The first volume contains an address entitled "Vito Volterra" delivered by G. Castelnuovo at the inaugural meeting of the Accademia after its reconstitution in 1946. This is followed by a detailed analysis of the scientific work of the great mathematician, given by Somigliana. Finally there is a very complete biography of Volterra, written by his friend and collaborator, J. Pérès.

The papers are arranged in chronological order, beginning with a paper on the potential of an ellipsoid, published in 1881 before

Volterra had attained the age of twenty-one, and ending with a paper on hereditary elasticity published in 1939–1940 when Volterra was nearly eighty. Most readers will find it fascinating to read the original presentations of the ideas and discoveries of Volterra which became later the basis for a great part of modern mathematical research. The work of Volterra is so well known and has been so often analyzed that it would be superfluous if not presumptuous for the present reviewer to give another detailed discussion. Let us therefore only indicate briefly some of the contents of the five volumes, grouped according to their subject.

The theory of functionals as a generalization of the idea of a function of several independent variables was developed by Volterra in a series of papers published since 1887 and was inspired by the problems of the calculus of variations. These papers initiated the modern theory of functional analysis. They attracted attention at once from the foremost mathematicians of his time. Actually the name “functional” was introduced later by Hadamard and has now replaced Volterra’s original nomenclature. In developing this theory Volterra already followed a principle which guided him through many discoveries and which he called the passage from the discrete to the continuous. It was this principle which he applied to his celebrated researches on integral equations of Volterra’s type. He considered heuristically the integral equations as a limiting case of a system of linear algebraic equations and then checked his final formulae directly. His procedure opened the way for Fredholm and Hilbert who, however, investigated the limiting process itself. Connected and somewhat preceding this work was Volterra’s investigation of what was called at that time the inversion of definite integrals.

About the same time, in 1892, Volterra published a series of fundamental memoirs on the integration of hyperbolic equations, especially concerning the equation of cylindrical waves. His method of integration, which can be roughly described as a Green’s method, yielded several results beyond the known results of Poisson and Kirchhoff. It opened the way to the investigations of Hadamard on Huygens’ principle and on the behavior of the wave equation in an even and odd number of dimensions. Volterra’s reduction of a Cauchy problem to an integral equation has influenced several modern methods of integration of hyperbolic equations.

Volterra was fascinated by the concept of multi-valued functions. In 1892 he gave a final solution to the problem of wave propagation from a center in a doubly-refracting medium. He observed that the solution previously given by Lamé contained many-valued functions,

a fact overlooked by S. Kowalewsky and therefore invalidating her conclusions. Let us note in passing that it was later found that many-valued functions have led other eminent mathematicians to erroneous results. However, in another domain, Volterra showed the importance of many-valued functions by developing the theory of dislocations in elastic solids which occupy a multiply-connected region of space. In this case the displacements may be many-valued functions while the stress remains single-valued.

In the third volume the reader will find some of Volterra's celebrated papers on integro-differential equations which he introduced in connection with his work on hereditary elasticity.

A considerable part of the work of Volterra in the latter part of his life was devoted to the applications of mathematics to biology. The subject of these investigations was mainly the study of biological associations of animals of different species living together. In other words he was interested in a mathematical theory of the "survival of the fittest." While there are today other methods of a stochastic nature, the work of Volterra still exerts a dominant influence on several modern and quite recent developments in mathematical biology.

Volterra's influence on the development of mathematics could make an attractive study for a historian of science. Hadamard publicly and enthusiastically acknowledged the powerful inspiration which he received from his older friend Volterra. Obviously this inspiration was felt in particular in Italy even in the work of the younger generation who could not have had personal contact with him.

This short review cannot be concluded without pointing out the excellence of the typographical work of the Accademia dei Lincei which produced in clear and attractive print these five volumes of nearly 3000 pages.

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