particular, Rudin constructs a closed ideal in $A(\Gamma)$ which is not self-adjoint, an interesting refinement of Malliavin's construction.

(Chapter VIII) How to extend properties of Taylor's series, or analytic functions in an open disc, when the group of integers $Z$ is replaced by another group? The question has been investigated by Arens, Helson, Hoffman, and Singer, when $Z$ is replaced by an ordered group. In the case of $Z^2$, for example, a Taylor series will be replaced by a Fourier series whose coefficients vanish in a half plane. The fundamental work concerning the $L^2$-theory is due to Helson and Lowdenslager, who succeeded in extending classical theorems of Szegö and Beurling. A theorem of the author, on Paley sequences, is given in this general context, as well as the most important results about conjugate functions ($L^p$-theory, starting from Helson's extension of M. Riesz's theorem).

(Chapter IX) What is the structure of the closed subalgebras in $L^1(G)$ or $A(\Gamma)$? A complete answer to the question, even when $\Gamma$ is discrete, is out of range (let us mention that the nice conjecture on p. 231 has been disproved since the book was published). Results of Wermer and Simon on maximal subalgebras are discussed, as well as the characterization of those groups $\Gamma$ such that $A(\Gamma)$ contains no proper closed separating subalgebra ("Stone-Weierstrass property"), discovered by Katznelson and the author.

The mere enumeration of the topics shows that the book is rich in material. Nevertheless, it is a pleasure to read. Two chapters (basic theorems in Fourier analysis, and structure of locally compact abelian groups) and appendices at the end, are intended to make it "self-contained," at the level of a graduate student. In the main part of the book, all the theorems are proved, and all proofs are complete, as far as the reviewer could notice. The author is known to be an excellent expositor, and he proves it once more by providing a considerable amount of information in less than three hundred pages without giving anywhere the impression of hurrying or pressing the reader. The printing is very good, and the book is as good looking as it is thorough.

J.-P. KAHANE

*Foundations of differential geometry.* By Shoshichi Kobayashi and Katsumi Nomizu. Interscience Tracts in Pure and Applied Math., No. 15. John Wiley and Sons, Inc., New York, 1963. 11+329 pp. $15.00.

Differential geometry has radically changed in recent years. An approach based on the theory of differential manifolds has replaced the classical tensor analysis (although the latter retains its usefulness as a computational tool), and has pointed the way towards many new and exciting problems. However, the lack of suitable books has hindered the development of the subject itself and prevented the interaction between geometry, analysis and mathematical physics that has been so fertile in the past.

This book is an effort to fill this pressing need. The authors state that it is intended to provide a "systematic introduction to differential geometry that will also serve as a reference book." As one would expect from the authors, both eminent geometers, there is much that is valuable, useful and definitive. However, this dual goal has, at least judged by this first volume, diluted the value of the book either as a graduate textbook, a reference work, or as material for self-study. This review will cover the book mainly for the beginner, say the mature mathematician trying to learn the essentials. (The book would be almost inaccessible to, say, a physicist interested in the possible applications to relativity.) It suffices to tell the expert that he will probably find the best features to be the complete and well-organized proofs of the numerous "folk theorems," the systematic and well thought out treatment of the theory of connections in principal fibre bundles, and the closing notes giving a skillful survey of the wide horizons.

Chapter 1 deals with the basic material concerning differentiable manifolds and Lie groups. The first section is most unfortunate: The authors squeeze into sixteen pages the concepts of manifolds, vector field, differential forms and their behavior under mappings. This is done by referring most of the proofs to Chevalley's "Lie groups," a notoriously difficult source. The reviewer has found in his teaching that the most difficult and important task for a student of the subject is to acquire facility and intuition for these basic concepts, and to learn to compute with them. Particularly to be regretted is that the authors pass up the opportunity to gather together the material, most of it truly fundamental, concerning completely integrable systems (involutive distributions) that is scattered in the literature. The next topics, tensor fields and Lie groups, are well done, although the authors do not explain how classical tensor analysis is obtained by specialization and remain on an excessively formal level. The closing material on fibre bundles is very good, isolating precisely what is needed in differential geometry.

Chapter 2 contains a detailed treatment of connections, curvature and holonomy group in principal bundles. §§1–6 define the basic notions and §§7–10 prove, in welcome detail, the general theorems relating holonomy and curvature. §11 concerns invariant connections. This chapter does not give a complete exposition of the original Ehresmann theory of connections; the greatest gaps are the lack of discussion of a connection as a field of horizontal subspaces for a maximal rank map, and the theory of Cartan connections.

Chapter 3, titled "Linear and affine connections," specializes the general theory to the most important case, presenting elegantly the standard material concerning parallel translation, geodesics and normal coordinates. A special feature is material clearing up the confusion between affine and linear connections, and showing the relation between the bundle and covariant derivative definitions.

Chapter 4 specializes further to Riemannian metrics and connections. The main theorems proved here are the Hopf-Rinow theorem concerning completeness and the de Rham decomposition theorem. It is regrettable that the reader does not see more nontrivial material concerning Riemannian geometry, such as the theory of submanifolds, Jacobi fields or the global properties of geodesics, but must wait until Volume 2. For example, later on, an overelaborate proof must be given of the fact that the sphere has constant curvature, because material on the second fundamental form is not yet available.

Chapter 5, titled "Curvature and space forms," presents only the barest introduction to these important subjects. For example, there is nothing at all concerning the geometric meaning of sectional curvature. The closing §4 is valuable, containing nontrivial information concerning flat affine connections. Chapter 6 deals with the group of automorphisms of an affine connection or Riemannian metric and its relation to curvature and holonomy. This closing chapter maintains a uniformly high standard, gathering together many fragments in an efficient way and demonstrating the advantages of the global methods over those of classical tensor analysis.

In summary, the book falls naturally in two parts, the first presenting in a definitive fashion the relation between curvature, holonomy and connections in a principal fibre bundle, the second introducing the reader to the most important branch of differential geometry, the theory of affine connections and Riemannian metrics. The most important topics (from the beginner's point of view) in this second part are done much more skillfully and readably in the first chapter of Helgason's *Differential geometry and symmetric spaces*. There are no exercises, almost no examples, and not the slightest attempt is made to present to the modern reader the marvelous geometric insights into Riemannian geometry pioneered by Cartan in his *Géom-*

*étrie des espaces de Riemann.* Only in the closing notes do the authors unbend and approach showing the beginner what differential geometry is all about; on the other hand there is not enough advanced material to really serve the expert as a reference book.

ROBERT HERMANN

*Introduction to knot theory.* By R. H. Crowell and R. H. Fox. Ginn and Co., Boston, Mass., 1963. 10+182 pp. $8.00.

It takes a bit of hunting to find any reference to the real world in most modern books on topology. The roots of knot theory however are in our physical world, so that it is not surprising that this introduction to knot theory places itself in the position of attacking a "practical" problem. What is surprising is the degree of practicality maintained throughout the book, for it is frequently the case that a mathematician's solution to a *real* question is useless from a pragmatic point of view.

The mathematician who is unfamiliar with knot theory, but familiar with topology will find in this book an emphasis which may be foreign to him, that is, given a knot (say by a photograph) the absorption of the contents of this book makes relatively easy the actual computation of invariants of the knot.

The mathematician who is unfamiliar with topology will find this book an excellent starting point. The juxtaposition of a theory with its applications makes for interesting and instructive reading. It is often very hard to understand a theorem in vacuo, and this book is so well knit that this unfortunate state of affairs is generally avoided.

It must be said that although the foregoing comments may seem to imply a rather special and simpleminded sort of mathematics being done in this work, such is very definitely not the case. Although the distinguishing of knots sounds innocent enough, the mathematics used is occasionally quite general, occasionally quite sophisticated, occasionally quite interesting, and always precise. That is not to say that the book is of a formal character; it is formal only when it needs to be, formality for its own sake is carefully avoided, a virtue which is unfortunately not universally agreed upon as such.

Reluctance to buckle down and *learn* the theory put forward in this book will not prevent the reader from profiting from a reading. There is often a good deal of informal discussion prior to making a definition or a proof, and this makes it possible to learn about the subject without wading through yards of notation. On the other hand this aspect of the presentation also makes it possible to better understand the details of the subject.