## BOOK REVIEWS

*Normed rings.* By M. Naimark. Noordhoff, Groningen, 1959. xvi +560 pp. $13.00.

The original Russian version of this work was published in Moscow in 1955 and now this present American edition has been with us about three years. Many reviews of the book have appeared in the meanwhile. Nevertheless, since this Bulletin, up to this moment, has not "taken notice" of this very unusual contribution to the organized literature, a few paragraphs seem to be in order.

There is little doubt about it. We have before us a work of outstanding importance. Its past has been brilliant and its future for a good many years will be equally distinguished. If one remembers correctly, the publishers advised us on the jacket that the book would be on the shelves of student and expert alike. This is almost an understatement. It is well written—lucidly so. And the translation is excellent. The proofs are given in detail and all background material is included. For example, the first chapter is a complete introductory treatise on topology, linear spaces, linear topological spaces, normed spaces, Hilbert space, and integration theory. And the chapter on groups devotes five pages (withal, in fine print) to the establishing of the necessary facts concerning the Haar measure. Thus nothing is taken for granted. The book is for the most part written so smoothly that one may heartily recommend it to students at the first year graduate level without fear that they will break a molar on some critical proof. And it is equally useful to the expert who wishes to view rapidly what has been done in certain directions or who wishes to find out how a proof goes after vainly searching around for a couple of hours (days?) without being able to reconstruct the obvious and well-known steps.

The development being axiomatic, the material is partially ordered. All necessary doctrines are moved forward more or less simultaneously in a deployment which is highly skilful but which at times makes one yearn for a more linear presentation. Thus in the early discussions on rings, the reader is instructed recurrently and at every turn in how to handle the case of a ring with and without identity, the commutative and noncommutative cases, the topological, normed, and complete cases. Rarer than one would desire is the golden phrase "from now on to the end of the chapter, all rings will be $\cdots$." This, I suppose is inevitable; c'est la nature de la bête. Later on when the beginner has disappeared from the scene, the multitude of types of rings will tax the expert (never the author). Here we find: symmetric rings, normed symmetric rings, regular rings, completely regular rings, reduced rings, regular norms, completely symmetric rings; to say nothing of dual rings, annihilator rings, Hilbert rings, and quite a few others. (Don't expect to find definitions here to all these delectable concepts.)

We have said before that the introductory chapter collects in a remarkably brief space the entire preliminary background for this material. It is much to be recommended to the student beginning in any branch of analysis and one feels that it could well be published separately with very little modification and with much benefit to the community. Of the seven remaining chapters, if one is to play favorites, this writer would choose Chapters VI and VII; the first on group rings (convolution algebras) and abstract harmonic analysis; the second on rings of operators in Hilbert space. In the first of these, the author carries the reader in 80 pages from the definition of a group (not even topological) over a chain of mathematical peaks including the Pontryagin duality theorem, Wiener's Tauberian theorem and the Peter-Weyl theory. Chapter VII develops brilliantly in 50 pages the Murray-von Neumann theory of factors including detailed examples of the various types except for type III. How straightforward all this looks now as compared to the original papers which we read over twenty years ago! It has certainly not been easy heretofore to find this theory readily available in a bona fide text. May one raise the question with the many members of the brilliant American school in operator theory as to why we have not had made available between hard covers all this material and more recent results too?

The last chapter treats the problem of the decomposition of a ring of operators into irreducible rings. This section, following very closely two papers of Tomita of 1953 and 1954, has not been universally accepted. In particular some of the results obtained do not seem to be valid in the generality claimed for them (the nonseparable case).

The various errors pointed out in Hewitt's review (Math. Reviews **19** (1958), 870–871) of the first Russian edition do not appear in the present rendition. However, the number of misprints which still remain is not negligible. Some twenty pages are devoted to a listing of papers of interest in the subject. Out of 178 authors or co-authors listed, 51 are Russian. Four pages are given over to a listing of notations. This section is of little value. The reader may find there (in case he still is in doubt) that $A \cap B$ stands for an intersection of sets and that $L^\infty$ is the set of all measurable essentially bounded functions. ($a \in A$ is also listed.) But no one will tell him where to look for the definition of $\bar{S}^4$, $\mathfrak{M}_f^{M'}$, or $C^r(\mathfrak{N})$.

The contributions of the Russian school to the theory of rings are outstanding. Indeed the architects of much of the theory are the triumvirs Gelfand, Naimark, and Shilov. How fortunate for us that one of these three has put down in extenso his essence in writing. One can only purloin Dostoyevsky's famous last words about the original Three Brothers and exclaim in admiration: "Hurrah for Naimark."

EDGAR R. LORCH

*Stationary processes and prediction theory.* By H. Furstenberg. Annals of Mathematics Study No. 44. Princeton Univ. Press, Princeton, N. J., 1960. 283 pp. $5.00.

This work is an elaboration of the author's doctoral dissertation at Princeton. The limitations of the classical prediction theory of stochastic processes are first discussed. In the light of this discussion a new prediction theory for single time-sequences is formulated. The ideas uncovered in the course of this development are shown to have interesting ramifications outside prediction theory proper. In the author's opinion the discussion of these offshoots, for which prediction is more of an "excuse" than a "reason" (p. 7), constitutes the most important part of the book. In this review we shall touch upon the critique, the new theory as well as the offshoots, but greater emphasis will be placed on the second topic in relation to the third both from considerations of space and the reviewer's predilections. We shall conclude the review with some general remarks on the work.

The book abounds with strange terminology, which has to be understood to get any insight into it. It is also rather complex in structure. In this review we have thus been obliged to state definitions and to indulge in an abridged and sometimes over-simplified exposition of the author's theory. It is hoped that this exposition will serve as a guide to the prospective reader of the book.

## I. LIMITATIONS OF CLASSICAL PREDICTION THEORY

It is well known that we are able to prognosticate the future in many realms in which strictly deterministic laws do not prevail. One scientific explanation of this ability is that such realms are governed by probabilistic laws in which the underlying probability measure is invariant under time-shifts. More precisely, underlying such a realm is a probability space $(\Omega, \mathcal{B}, P)$, and a $P$-measure-preserving transformation $T$ on $\Omega$ onto $\Omega$. We are interested in some $\mathcal{B}$-measurable function $f$ on $\Omega$ or what amounts to the same thing, in a stationary