BOOK REVIEWS

Science awakening. By B. L. van der Waerden. English translation by Arnold Dresden, with additions of the author. Oxford University Press, New York, 1961. 306 pp., 133 fig., 28 plates. $7.50.

Some people, even mathematicians, may wonder why so much time is spent on digging out and on deciphering clay tablet texts with clumsy solutions of trivial quadratic equations, or what is the fun of Theaetetus' horrible theory of irrationalities, and whether we can still learn anything from Archimedes' old-fashioned integration methods. But with the same right, one might ask whether it pays to take up a mathematical problem which has been solved once, and to solve it once more in another way. Mach defined science as economy of thought, though actually it is not a virtue of the scientist to take it easy, but rather to look for problems which can stir up his curiosity. If there is any need for justifying the history of science, it would be its extraction from human curiosity and its power to satisfy this same curiosity. Perhaps in the history of science oddities and deadlocks have been more frequent than trail-blazing ideas. In a faithful image of our scientific past both of them are indispensable, like the background and the theme of a painting. But as soon as we concentrate on the main line, we are struck by the sensation that ancient science is more modern than one might expect. Apollonius' Conics could have been written the day before Descartes invented analytic geometry; Dedekind could have explained his theory of real numbers in a letter to Eudoxus, and I wonder whether today we are not taking up the linguistic-ontological problems which disturbed the sophists, Plato, and Aristoteles and led them to their philosophical theories, just at the point where they stood in the 4th century B.C. I admit such arguments cannot fundamentally justify the history of science. I firmly believe that as human beings we have the duty not only to know our place in the world of today, but also in the chain of the heritage we have received from our ancestors and which we have to hand down to posterity.

This has been to explain why twenty-odd years ago a famous mathematician turned to the history of mathematics and astronomy. It should be added that when doing so he forsook neither creative mathematics nor the spirit of mathematical research. Van der Waerden's zeal for clear and thorough understanding, meticulous exactness and lucid explanation and his intuitive imagination are no less striking in his historical work than they have been in his mathematical papers and textbooks, and his mathematical many-sidedness has its counterpart in his approach to history. In the course of the years he has published a long series of specialized research papers in the history of sciences, painstaking analyses of texts and tables, and a number of major and more synthetic papers, mainly on Pythagorean mathematics, musical theory, and astronomy. His historical analysis of some parts of Euclid's elements is an outstanding example of a rare blend of mathematical and historical understanding.

It is not useless to stress these facts. Though the present work is not at all a collection of van der Waerden's own discoveries, it could never have been written by a mere compilator. Historical research means a rigorous training in criticism, and there is no better way to get a feeling for the weight of historical arguments, and for the spirit of some remote epoch than the careful reading and collating of original texts.

I mentioned that the present work is neither a collection of historical discoveries nor a compilation. It does not claim any encyclopedic character either. In the preface van der Waerden set his principal goals, which are not many, but which are lofty enough to be promising. His aim is to explain "how Thales and Pythagoras took their start from Babylonian mathematics but gave it a very different, a specifically Greek character; how, in the Pythagorean school and outside, mathematics was brought to a higher and ever higher development and began gradually to satisfy the demands of stricter logic; how through the work of Plato's friends Theaetetus and Eudoxus, mathematics was brought to the state of perfection, beauty and exactness, which we admire in the elements of Euclid. We shall see moreover that the mathematical method of proof served as a prototype for Plato's dialectics and for Aristotle's logic." Less than one third of the book (the first three chapters) is dedicated to pre-Greek mathematics (with a digression on Hindu and European medieval methods in the history of number systems and computing), and less than the last third comprises Greek mathematics from Euclid onwards.

This does not mean that Euclid's elements are neglected, but they are dealt with as a source of historical information on pre-Euclidean mathematics rather than as a self-contained work of mathematical literature. Eudoxus' authorship of Euclid's books V and XII (epsilontics), and Theaetetus' of book X (classification of irrationalities) and XIII (regular bodies) are generally accepted today, and all arguments for these attributions are brought together in van der Waerden's book. Euclid II is probably a Pythagorean adaptation of Babylonian algebra to Greek geometric methods. A very old Pythagorean fragment (the "even and odd") was discovered by Becker in Euclid's IX, 21–34, 36. After van der Waerden's marvellous analysis of VII and VIII it may be taken for granted that VII is an anonymous Pythagorean textbook of number theory and that the quite chaotic VIII, which is closely related to musical theory, has to be ascribed to Archytas. (The historical layers in the other Euclidean books are still uncovered. XI (solid geometry) might be relatively easy, whereas I, III, IV ask for a more philological analysis.)

The idea that Euclid was rather an editor than a creative mathematician is quite recent. Much attention has been paid in the last few decades to apocryphic material, rejected by Euclid, because it belonged to competing theories or because it did not fit into the frame of the Elements or other Euclidean works. Any allusion to mathematics in Plato's dialogues, any quotation of a byzantinic commentator which is linked to ancient mathematical sources, may shed new light on the course of history. The greater part of this material is still controversial. Out of this great variety of sources van der Waerden confines himself to the few which nowadays are clearly understood. He refrains from bothering the reader with more material than he can use and with hypotheses which cannot be proved by something that is closely akin to a mathematical proof.

It should be kept in mind that van der Waerden's book, though useful for scholars in history of science as an organization of dispersed material on selected topics, is written for the general reader who has a basic knowledge of mathematics and who would like to learn about the way in which this science has originated. Mathematics is put into a frame of political and cultural history, the utterly most striking feature of which is a large number of judiciously chosen beautiful plates.

The book first appeared in Dutch (1950). Translating it must have been enormous work. The book has had to be rewritten in English to reflect van der Waerden's completely plain and wholly unsophisticated Dutch style. I have compared the two versions in many places. None of them gave rise to the slightest criticism. Dresden has done the utmost an honest translater can do and much more than most of them are able to do.

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