BOOK REVIEWS

Solution of equations and systems of equations. By A. M. Ostrowski. Academic Press, New York, 1960. 9+202 pp. $6.80.

The book is the outgrowth of a series of lectures given at the American University, Washington, D. C., during the summer of 1952 at the invitation of the National Bureau of Standards. Much of the material contained in the course was unpublished. In preparing the book, the course was completely rewritten and augmented by additional new material.

Of primary concern is the solution of a single equation $f(x)=0$. The following methods are considered: the use of inverse interpolation including the use of a single point, the use of two points (regula falsi), and the use of an arbitrary number of points; the inversion of Taylor series involving the use of two terms (the Newton-Raphson method), and the use of an arbitrary number of terms; and the use of iteration procedures of the form $x_{n+1}=\psi(x_n)$, where $\psi(x)$ is a function such that $x = \psi(x)$ if and only if $f(x) = 0$. The use of the function $w=(\alpha x+\beta)/(\gamma x+\delta)$ is also considered, where α, β, γ, δ are determined so that for three values of x, the values of w agree with $f(x)$. In addition, an analogue of the Newton-Raphson method is given for multiple roots. In each case the author gives sufficient conditions for convergence and for monotone convergence, estimates of the rapidity of convergence, and the "efficiency index" which depends on the rapidity of convergence and the amount of work per iteration.

Preparatory to considering the solution of n equations with n unknowns the author develops some of the theory of norms of vectors and matrices and of the convergence and divergence of infinite products of matrices. He then obtains conditions for certain iterative procedures and for given solution vectors of the equation system so that, starting sufficiently close to the solution vector, the vectors obtained will converge to the solution.

There are 11 appendices to which 70 pages are devoted. The following topics are covered: continuity and relative continuity of the roots of algebraic equations as functions of the coefficients; explicit formulas for the nth derivative of the inverse function; an analogue of the regula falsi for two equations in two unknowns; Steffenson's improved iteration rule; the Newton-Raphson algorithm for quadratic polynomials; some modifications and improvement in the Newton-Raphson method; rounding-off in inverse interpolation; accelerating iterations with superlinear convergence; determining the roots of

$f(z) = 0$ from the coefficients of the development of $1/f(z)$; and the continuity of the eigenvalues of a matrix as functions of the coefficients. There are five pages of bibliographical notes and an index.

The treatment is mathematically rigorous and clearly written. The book is an essential reference to anyone intending to study or to do research on the solution of equations and systems of equations. Many proofs of important results are given which are either not readily available or not clearly explained elsewhere. Examples include: the derivation of the remainder term in polynomial interpolation (Chapter 1); Darboux's Theorem on values of $f'(x)$ which makes it practical to consider methods based on inverse interpolation for solving $f(x) = 0$ (Chapter 2); convergence theorems for the various methods considered; the analysis of the rapidity of convergence of methods by means of linear difference equations (Chapter 12); norms of vectors and matrices (Chapter 15); and the continuity of the roots of equations as functions of the coefficients and of the eigenvalues of matrices as functions of the elements.

As the author states in the preface, the treatment is far from complete. One important method, namely that due to Muller [MTAC **10**(1956), 208–215], which has been extensively used as a basis for computer programs, is not considered. The method is rejected, in Chapter 11, on what appears to be superficial grounds, namely, that at each iteration one is required to choose between two roots of the interpolating quadratic polynomial. For the method considered instead, namely that involving inverse interpolation with the function $w = (\alpha x + \beta)/(\gamma x + \delta)$, if the function $f(x)$ assumes real values for real x, then the use of real starting values will not lead to complex roots. The same drawback holds for regula falsi and the Newton-Raphson method. On the other hand, Muller's method will lead to complex roots even for real starting values. In general, there seems to be insufficient attention given in the book to the determination of complex roots.

While a number of numerical examples are given, there is little comparative discussion of the methods from the following point of view; given a problem, how should one go about selecting an appropriate method? In other words, can one prescribe a series of steps which one could follow, or which a computer could follow, and which in a given case would either lead to the solution or else indicate that none of a given set of methods is applicable? This would be worthwhile even for a restricted class of problems such as polynomial equations. These criticisms are not intended to detract from the importance and usefulness of the book but simply to indicate that a great deal of additional work remains to be done on the solution of equations and systems of equations.

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Topology. By John G. Hocking and Gail S. Young. Addison-Wesley, Reading, Mass., 1961. 9+374 pp. $8.75.

This book is designed as a text for a one year first course in topology. Three chapters on general topology and a chapter on homotopy theory constitute the proposed first semester's course, four chapters on algebraic topology the second term's program. The authors have adopted a policy of including by mention or brief description many topics not covered extensively, with the object of providing orientation for further study.

The first two chapters proceed in more-or-less standard fashion with the study of the notion of a topology and the elementary properties of topological spaces: compactness, connectedness, separation and so forth. There is concordantly a discussion of metric spaces. The scope may be suggested by the principal theorems, viz. the Tychonoff compactness theorem, the Tietze extension theorem, the Urysohn metrization theorem and the Baire category theorem. Orientational material, treated perfunctorily and in general without proofs, includes function spaces, uniform structures, topological groups, paracompactness, the Smirnov metrization theorem, inverse limits.

The third chapter contains a more intensive study of compacta than is fashionable nowadays, with a proof of the Hahn-Mazurkiewicz theorem and discussion of monotone-light factorization and indecomposability. There is also a nod towards dimension theory.

Chapter four compresses a remarkably large amount of homotopy theory into 43 pages: homotopy of maps, Borsuk's homotopy extension theorem, essentiality, absolute homotopy groups, knot theory, covering spaces, homotopy local connectedness. Some of these subjects are of course only briefly mentioned. The knot theory is purely descriptive; covering spaces are honored by a definition and a few theorems stated without proof. But it may be fair to say that homotopy theory suffers from the compression. The treatment seems too dense to be readable at this level, and the choice of theorems proved is ill-calculated to give a coherent picture of the structure of the theory.

The next two chapters are devoted to the machinery of homology theory of simplicial complexes, i.e. to the geometry of polytopes and the construction of chain, cycle, boundary and homology groups. Chapter seven discusses relative homology and cohomology, and in-