

terior of a single star-shaped obstacle and for any solution which satisfies the boundary condition $u=0$ with Cauchy data at $t=0$ vanishing outside a sphere S , that the energy contained in sphere D is less than⁷ $\text{const. } E/t$, where E is the total energy and the value of the constant depends only on the radii of S and D . We wish to point out that such a quantitative result about energy decay can hold only if the obstacle satisfies the following geometric condition: No two boundary points of it form a segment exterior to the obstacle and perpendicular to it at both endpoints. For a narrow high frequency beam directed along such a segment would be reflected back and forth for a length of time proportional to the reciprocal of wave length.

If one regards the presence of the obstacle as a perturbation of free space and applies the usual scattering theory formalism,⁸ then it follows from Huygens' principle in free space that the so-called wave operators exist, and it follows further from Theorem II that they are unitary. This proves

THEOREM III. *The perturbed and unperturbed problems are unitarily equivalent; in particular, the spectrum of the wave equation generator is unaffected by the presence of obstacles.*

Added in proof. Recently we have succeeded in deriving Theorem II without making use of Huygens' principle, by using solutions which are superpositions of plane waves of finite width. Thus the results of this note can be extended to a large class of hyperbolic equations.

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⁷ In a more recent paper, Morawetz has shown that energy decays like $1/t^2$.

⁸ See S. T. Kuroda, *Perturbation of continuous spectra by unbounded operators*. I, J. Math. Soc. Japan 11 (1959), 247-262; or J. M. Jauch, *Theory of the scattering operators*, Helv. Phys. Acta 31 (1958), 127-158.

ADDENDUM TO ON THE EIGENVALUE OF POSITIVE OPERATORS¹

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Two important assumptions on the operator P in the Theorem stated in this note were omitted by a clerical error. They are

- (a) the operator P is *positive* ($Pf \geq 0$ if $f \geq 0$).
- (b) the number α is of absolute value *one*.

These assumptions are stated in the informal discussion at the beginning of the note, and are used throughout in the proof.

¹ See volume 67, no. 6 (November 1961) pp. 556-558 of this Bulletin.