

uniform, if and only if it is unitarily invariant, i.e., $\alpha(UAV^*) = \alpha(A)$ for $A \in \mathfrak{K}$ and any unitary operators U, V on \mathfrak{S} . Every unitarily invariant crossnorm on \mathfrak{K} can be defined in a simple manner in terms of a symmetric gauge function on the linear space \mathfrak{L} of infinite sequences of real numbers having only a finite number of nonzero terms. Let $\mathfrak{K}(\alpha)$ denote the normed linear space obtained by defining a unitarily invariant crossnorm α on \mathfrak{K} , and let \mathfrak{K}_α denote the metric completion of $\mathfrak{K}(\alpha)$. It turns out that \mathfrak{K}_α may be identified with a minimal norm ideal of \mathfrak{A} and that every minimal norm ideal can be obtained in this way.

Some of the material (e.g., the theorems $\mathfrak{C}^* = (\tau c)$ and $\mathfrak{C}^{**} = \mathfrak{A}$, the relationship between the unitarily invariant crossnorms on \mathfrak{K} and the symmetric gauge functions on \mathfrak{L}) may be already found in the author's earlier book, *A theory of cross-spaces* (Princeton University Press, 1950), but their inclusion makes the present book self-contained. This monograph, written in great clarity, is an excellent exposition of the author's study of spaces and ideals of completely continuous operators. It should be of interest to those working on theory of operators or Banach algebras.

KY FAN

Analytic functions. By R. Nevanlinna, H. Behnke, H. Grauert, L. V. Ahlfors, D. C. Spencer, L. Bers, K. Kodaira, M. Heins, and J. A. Jenkins. Princeton, Princeton University Press, 1960. 7+197 pp. \$5.00.

At first glance it would appear that this book has no other motivation than the transcription of the principal addresses of the 1957 Conference on Analytic Functions. This would put it in the same category as the 1953 Brussels Colloquium and reports of other colloquia. A glance through the table of contents reveals an article on complex spaces, two articles on the moduli of Riemann surfaces, an article on perturbation of structure and others. It seems to be a random collection of papers on function theory.

If this were the case, a review here of this book would be as ludicrous as a review of vol. 71 of the Transactions. But such is not the case. The book stands as an exposition of complex analysis in the last decade and treats well and fully the lines of research developed in that period. The majority of articles meets the needs of those with an outside interest in complex analysis who are curious to understand the new results and techniques. More important, the new student of complex analysis is presented with the tools and bibliography which he needs and cannot find elsewhere. From the point of view then, of

general information, this book differs from the journals and other colloquia volumes, and it is because of this that it is important and merits consideration here.

Surely, from this point of view the article of Behnke and Grauert is the most outstanding and most needed. It is the only modern exposition of several complex variables. After an introduction to the inevitability of the discovery of complex spaces (from the point of view of analytische gebilde) and a clear definition of such, the authors discuss various ways of altering these in the hope of arriving at more pleasant objects. There follows a description of analysis on holomorphically complete spaces; the role of plurisubharmonic functions; and a final section on complex fiber bundles. Throughout are illustrations of unexpected pathology, and descriptions of what can be said despite it. The only fault with the article is its omissions. The work of Cartan and Serre on analytic varieties is barely touched upon, and the normalization theorem of Oka is not mentioned. The facts and problems concerning the envelope of holomorphy of a non-holomorphically complete complex space are not treated.

The articles by Bers and Ahlfors faultlessly give an introduction to the problem of moduli of Riemann surfaces. Bers introduces the concepts of quasiconformality and generalized Beltrami equations and beautifully develops the relation between the two; making clear the role to be played by these in the problem of moduli. After developing Fuchsian groups as defining Riemann surfaces, he uses these to obtain the number of moduli of a compact surface. This is so done that when Teichmüller's theorem is stated it is not so mysterious and is easily proved on the basis of existence theorems for the Beltrami equations. Ahlfors gives the Teichmüller space a complex analytic structure so that the period matrix of a surface is a holomorphic function. His approach is different from that of Bers but as clear and elegant in stating the problem and effecting the solution. Our only regret is that the research which has been done since 1957 could not also have been reported here.

The remainder of the articles do not succeed as these do. Spencer gives a clear but difficult definition of structure and deformation, and fails to impart some feeling for the methods of their study. Heins gives a beautiful introduction to the study of conformal mappings and relations with mappings of rings of functions of Riemann surfaces, but his discussion of asymptotic spots and Lindelöfian maps lacks the exposition that the first sections contain.

It is (perhaps) unfair to make these criticisms (which bear no necessary relation to the authors' intent) without reiterating that we have

considered this book as it may affect a large audience rather than a particular one. As such it is an important contribution to the exposition of modern mathematics. It is indeed time for more such books, in the same field as well as others.

The references (p. 54) to Rauch are:

[6, 7] H. E. Rauch, *On the transcendental moduli of algebraic Riemann surfaces. On the moduli of Riemann surfaces*, Proc. Nat. Acad. Sci., U.S.A. **41** (1955), 42–48, 236–238. The references [64], [65], p. 44 come from the second paragraph on p. 40. [64] should read: cf. Cartan [60]. The Oka linking method is mentioned often in the same article. The reference [4] should be given.

HUGO ROSSI

Time series analysis. By E. J. Hannan. Methuen's Monographs on Applied Probability and Statistics, 1960, viii+152 pp. \$3.50.

This small monograph deals with statistical inference where the basic model is a one-dimensional discrete-parameter process $y_t = m_t + x_t$ with x_t a stationary residual and m_t a mean value term. The book assumes a background in mathematics and statistics at the level of H. Cramér's text "Mathematical Methods of Statistics."

The first chapter (The Spectral Theory of Discrete Stochastic Processes) discusses the probability structure of the processes considered. The concept of a stationary process is introduced. What the author terms circularly defined processes are then introduced. These are stationary processes with parameter set the integers mod n . The second order moment structure of these processes can be simply given in terms of circulant matrices. The author often obtains results for stationary processes by suggesting a circularly defined process as an approximation, deriving the result for these simple processes using circulant matrices and then getting the analogous conclusion for stationary processes by a formal limiting procedure. In particular, the spectral representation for stationary processes is obtained in this manner. A rigorous proof of the spectral representation is given in an appendix. Basic results in the prediction problem for stationary processes are cited but not derived. Kolmogorov's formula for the one-step prediction error is then used to derive the asymptotic distribution of eigenvalues of the covariance matrix. It is a pity that the basic analytic work of G. Szegő that is fundamental in this context is not mentioned.

The second chapter (Estimation of the Correlogram and of the Parameters of Finite Parameter Schemes) contains a derivation of a weak ergodic theorem that is based on the spectral representation of