BOOK REVIEWS

Norm ideals of completely continuous operators. By Robert Schatten. Ergebnisse der Mathematik und ihrer Grenzgebiete, Neue Folge, Heft 27. Berlin-Göttingen-Heidelberg, Springer-Verlag, 1960. viii +81 pp. DM 23.60.

Let $\mathfrak{H}$ be an infinite-dimensional complex Hilbert space, and let $\mathfrak{A}$ be the algebra of all bounded operators on $\mathfrak{H}$. An ideal (two-sided) $\mathfrak{T}$ of $\mathfrak{A}$ is made into a normed linear space by introducing a norm α on $\mathfrak{T}$. In general, the norm $\alpha(A)$ of $A\in\mathfrak{T}$ does not coincide with the bound $\|A\|$ of the operator A. A norm α is a *crossnorm*, if $\alpha(A)=\|A\|$ for all operators $A\in\mathfrak{T}$ of rank 1. α is *uniform*, if $\alpha(XAY)\leqq\|X\| \cdot\|Y\|\cdot\alpha(A)$ for all $A\in\mathfrak{T}$ and X, $Y\in\mathfrak{A}$. An ideal $\mathfrak{T}$ of $\mathfrak{A}$ is called a *norm ideal*, if there is defined on $\mathfrak{T}$ a uniform crossnorm with respect to which $\mathfrak{T}$ is a Banach space. As indicated by its title, this monograph is a study of norm ideals of $\mathfrak{A}$ formed by completely continuous operators.

After a preliminary discussion of completely continuous operators, the main part of the book begins with some general theorems on ideals of $\mathfrak{A}$: Calkin's theorem that, in case of separable $\mathfrak{H}$, every proper ideal of $\mathfrak{A}$ is contained in the ideal $\mathfrak{C}$ of all completely continuous operators; Kaplansky's theorem that if a left ideal of $\mathfrak{A}$ formed by completely continuous operators is closed in the uniform topology and annihilates only the nullvector, it must coincide with all of $\mathfrak{C}$.

Next, the reader is introduced to two well-known ideals of $\mathfrak{A}$: the Schmidt class (σc) and the trace class (τc). In a natural way, uniform crossnorms σ, τ are defined on (σc), (τc) respectively by $\sigma(A) =(\sum_j\|Ax_j\|^2)^{1/2}$ and $\tau(A)=\sum_j((A^*A)^{1/2}x_j,\ x_j)$, where $\{x_j\}$ is a complete orthonormal system in $\mathfrak{H}$. With these respective norms σ and τ, (σc) and (τc) are norm ideals of $\mathfrak{A}$ formed by completely continuous operators. The Banach space (τc), with τ as norm, may be interpreted as the conjugate space $\mathfrak{C}^*$ of the Banach space $\mathfrak{C}$ of all completely continuous operators, when the operator bound is taken as the norm in $\mathfrak{C}$. The Banach space $\mathfrak{A}$ of all bounded operators on $\mathfrak{H}$, again with the operator bound as norm, may be interpreted as the second conjugate space $\mathfrak{C}^{**}$ of $\mathfrak{C}$. But $\mathfrak{C}$ is not the conjugate space of any Banach space, when $\mathfrak{H}$ is infinite-dimensional.

The remaining part of the book deals mainly with the problem of determining all minimal norm ideals of $\mathfrak{A}$. A norm ideal is *minimal*, if none of its proper subspaces is a norm ideal. Let $\mathfrak{R}$ be the ideal of $\mathfrak{A}$ formed by all operators of finite rank. A crossnorm α on $\mathfrak{R}$ is uniform, if and only if it is unitarily invariant, i.e., $\alpha(UAV^*)=\alpha(A)$ for $A\in\mathfrak{R}$ and any unitary operators U, V on $\mathfrak{H}$. Every unitarily invariant crossnorm on $\mathfrak{R}$ can be defined in a simple manner in terms of a symmetric gauge function on the linear space $\mathfrak{L}$ of infinite sequences of real numbers having only a finite number of nonzero terms. Let $\mathfrak{R}(\alpha)$ denote the normed linear space obtained by defining a unitarily invariant crossnorm α on $\mathfrak{R}$, and let $\mathfrak{R}_\alpha$ denote the metric completion of $\mathfrak{R}(\alpha)$. It turns out that $\mathfrak{R}_\alpha$ may be identified with a minimal norm ideal of $\mathfrak{A}$ and that every minimal norm ideal can be obtained in this way.

Some of the material (e.g., the theorems $\mathfrak{C}^*=(\tau c)$ and $\mathfrak{C}^{**}=\mathfrak{A}$, the relationship between the unitarily invariant crossnorms on $\mathfrak{R}$ and the symmetric gauge functions on $\mathfrak{L}$) may be already found in the author's earlier book, *A theory of cross-spaces* (Princeton University Press, 1950), but their inclusion makes the present book self-contained. This monograph, written in great clarity, is an excellent exposition of the author's study of spaces and ideals of completely continuous operators. It should be of interest to those working on theory of operators or Banach algebras.

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Analytic functions. By R. Nevanlinna, H. Behnke, H. Grauert, L. V. Ahlfors, D. C. Spencer, L. Bers, K. Kodaira, M. Heins, and J. A. Jenkins. Princeton, Princeton University Press, 1960. 7+197 pp. $5.00.

At first glance it would appear that this book has no other motivation than the transcription of the principal addresses of the 1957 Conference on Analytic Functions. This would put it in the same category as the 1953 Brussels Colloquium and reports of other colloquia. A glance through the table of contents reveals an article on complex spaces, two articles on the moduli of Riemann surfaces, an article on perturbation of structure and others. It seems to be a random collection of papers on function theory.

If this were the case, a review here of this book would be as ludicrous as a review of vol. 71 of the Transactions. But such is not the case. The book stands as an exposition of complex analysis in the last decade and treats well and fully the lines of research developed in that period. The majority of articles meets the needs of those with an outside interest in complex analysis who are curious to understand the new results and techniques. More important, the new student of complex analysis is presented with the tools and bibliography which he needs and cannot find elsewhere. From the point of view then, of