CORRECTION TO "SPACES OF RIEMANN SURFACES AS BOUNDED DOMAINS"[1]

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Communicated May 31, 1961

In [3] I sketched a proof of the theorem: *Every Teichmüller space* $T_{g,n}$ *is a bounded domain in complex number space.*

This proof is invalid since Lemma B is false. The error in the argument occurs on page 101, lines 8–12. The theorem is nonetheless true. A complete proof will appear elsewhere; a brief outline follows. The same proof was found, simultaneously and independently, by Lars V. Ahlfors.

Let G be a Fuchsian group without elliptic elements and with the unit circle as a limit circle. Denote by U the unit disc and by V the domain $1<|z|\leqq\infty$. The Riemann surfaces $S=V/G$ and $\bar{S}=U/G$ are mirror images of each other.

Let M be the set of complex-valued measurable functions $\mu(z)$ such that $|\mu(z)|\leqq k(\mu)<1$, $\mu\equiv 0$ in U, and $\mu(z)d\bar{z}/dz$ is invariant under G. For $\mu\in M$ let $z\rightarrow w^{\mu}(z)$ be the homeomorphism of the plane onto itself which satisfies the Beltrami equation $w_{\bar{z}}=\mu w_z$ and is normalized by the conditions $w^{\mu}(0)=0$, $w^{\mu}(1)=1$. Then $G^{\mu}=w^{\mu}G(w^{\mu})^{-1}$ is a discontinuous group of Möbius transformations and $S^{\mu}=w^{\mu}(V)/G^{\mu}$ a Riemann surface. Also, w^{μ} defines a quasiconformal mapping f^{μ} of S onto S^{μ} and thus a point in the Teichmüller space $T(S)$; all points in this space can be so obtained. We say that μ and ν are equivalent if they define the same point in $T(S)$, i.e. if S^{μ} is conformal to S^{ν} and f^{μ} homotopic to f^{ν}. This is so if and only if there is a Möbius transformation C such that $C(w^{\mu}(z))=w^{\nu}(z)$ in U (cf. [2]).

Holomorphic quadratic differentials on $\bar{S}$ may be represented by G$-$automorphic forms of weight (-4) in U, i.e. by holomorphic functions $\phi(z)$, $z\in U$, with $\phi(z)dz^2$ invariant under G. We define the norm $\|\phi\|$ to be the supremum of $\lambda|\phi|$ where $\lambda(z)=(1-|z|^2)^2$. The quadratic differentials of finite norm form a complex Banach space B.

For $\mu\in M$ the function w^{μ} is holomorphic in U and so is its Schwarz derivative ϕ^{μ}; note that ϕ^{μ} depends only on the equivalence class $[\mu]$ of μ. One verifies directly that $\phi^{\mu}(z)dz^2$ is G-invariant, and by a theorem of Nehari [4] we have that $\|\phi^{\mu}\|\leqq 6$. Knowing ϕ^{μ} we may recon-

[1] This paper represents results obtained at the Institute of Mathematical Sciences, New York University, under the sponsorship of the Office of Ordnance Research, U. S. Army, Contract No. DA-30-ORD-2153. Reproduction in whole or in part is permitted for any purpose of the United States government.

struct w^μ as the quotient of two solutions of the ordinary differential equation $2\kappa'' = \phi\kappa$. Hence $[\mu] \rightarrow \phi^\mu$ is a one-to-one mapping of $T(S)$ onto a bounded set $W \subset B$. This mapping is holomorphic in the following sense: if $\mu \in M$ depends holomorphically on complex parameters, so does $\phi^\mu(z)$, for every fixed $z \in U$ (cf. [1]).

Assume now that G is finitely generated. Then S is obtained from a closed surface of genus g by removing $n \geqq 0$ points with $3g-3+n>0$, $\dim B = 3g-3+n$, and $[\mu] \rightarrow \phi^\mu$ is a holomorphic homeomorphism of $T(S) = T_{g,n}$ onto W. Since $\dim T_{g,n} = \dim B$, W is a domain.

References

1. Lars V. Ahlfors and Lipman Bers, *Riemann's mapping theorem for variable metrics*, Ann. of Math. vol. 72 (1960) pp. 385–404.

2. Lipman Bers, *Simultaneous uniformization*, Bull. Amer. Math. Soc. vol. 66 (1960) pp. 94–97.

3. ———, *Spaces of Riemann surfaces as bounded domains*. ibid. vol. 66 (1960) pp. 98–103.

4. Zeev Nehari, *The Schwarzian derivative and schlicht functions*, ibid. vol. 55 (1949) pp. 545–551.

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