BIBLIOGRAPHY

1. M. G. Barratt, *Truck groups.* I, Proc. London Math. Soc. vol. 5 (1955) pp. 71–106.
2. R. Bott, *The stable homotopy of the classical groups*, Ann. of Math. vol. 70 (1959) pp. 313–337.
3. I. M. James, *The intrinsic join: a study of the homotopy groups of Stiefel manifolds*, and *Cross-sections of Stiefel manifolds*, Proc. London Math. Soc. vol. 8 (1958) pp. 507–636 and pp. 536–547.
4. ———, *Whitehead products and vector fields on spheres*, Proc. Cambridge Philos. Soc. vol. 53 (1957) pp. 817–820.
5. H. Toda, *Composition methods in homotopy groups of spheres*, to be published in Annals of Mathematics Studies, Princeton University.
6. G. W. Whitehead, *A generalization of the Hopf invariant*, Ann. of Math. vol. 51 (1950) pp. 192–237.
7. J. H. C. Whitehead, *On the groups $\pi_r(V_{n,m})$ and sphere-bundles*, Proc. London Math. Soc. vol. 48 (1944) pp. 243–291.

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COMPACT KAEHLER MANIFOLDS WITH POSITIVE RICCI TENSOR

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The purpose of the present note is to announce the following:

THEOREM 1. *A compact Kaehler manifold with positive definite Ricci tensor is simply connected.*

We say that the first Chern class of a compact Kaehler manifold is positive definite if it can be represented by a real closed (1, 1)-form which is positive in the sense of Kodaira [2]. The first Chern class of a manifold satisfying the assumption in Theorem 1 is necessarily positive definite. Theorem 1 follows from the following two theorems.

THEOREM 2. *If the first Chern class of a compact Kaehler manifold M is positive definite, then the fundamental group of M has no proper subgroup of finite index.*

THEOREM OF MYERS. *The fundamental group of a compact Riemannian manifold with positive definite Ricci tensor is finite* [3].

Theorem 2 can be proved by Kodaira's Vanishing Theorem and by the Riemann-Roch Theorem of Hirzebruch. Let g_p be the dimension of the space of holomorphic p-forms on M. Then $\chi(M) = \sum_{p=0}^{n} (-1)^p g_p$, where $n = \dim_C M$, is called the arithmetic genus of M. If M is algebraic, then $\chi(M)$ is given as the integral over M of a polynomial in Chern classes c_i of weight n, polynomial depending only on n, not on M [1]. From this follows that if M^* is a k-fold covering space of M, then $\chi(M^*)=k\cdot\chi(M)$. On the other hand, if the first Chern class is positive definite, then $g_p=0$ for $1\leqq p\leqq n$ [2] and, hence, the arithmetic genus is 1. If the first Chern class of M is positive definite, so is the first Chern class of M^*. Hence, $\chi(M^*)=\chi(M)=1$, proving that $k=1$.

Note that Theorem 2 can be rephrased as follows. If the first Chern class of M is positive definite, then every holomorphic transformation of finite period has fixed points.

In view of the fact that we know no example of a compact Kaehler manifold with positive definite first Chern class whose Ricci tensor is not positive definite, we conjecture that M is simply connected under the assumption of Theorem 2.

References

1. F. Hirzebruch, *Neue topologische Methoden in der algebraischen Geometrie*, Ergebnisse der Mathematik und ihrer Grenzgebiete, vol. 9, Berlin, Springer, 1956.

2. K. Kodaira, *On a differential-geometric method in the theory of analytic stacks*, Proc. Nat. Acad. Sci. U.S.A. vol. 39 (1953) pp. 1268–1273.

3. S. B. Myers, *Riemannian manifolds with positive mean curvature*, Duke Math. J. vol. 8 (1941) pp. 401–404.

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