

fundamental quantities and relations. Results include the Hilbert-Schmidt Theorem and the Schmidt (eigenfunction expansion) formula for a solution.

In addition to the appendix on Lebesgue integration, a short appendix on reduction of a quadratic form to a canonical form is given. Besides an adequate index, a handy list of theorem titles and their page references is provided.

This book is a good text for an introductory course in integral equations at the advanced undergraduate or first-year graduate levels or for self-study. Although a few examples and exercises are provided, additional examples, applications and exercises should be provided if used as a text. The logical order and motivation for the theorems make for rapid understanding.

JOHN H. BARRETT

General theory of Banach algebras. By Charles E. Rickart, New York, Van Nostrand, 1960. 11+394 pp. \$10.50.

A report on Soviet mathematics, written by Professor J. P. LaSalle and summarizing the result of a year's study by a panel organized by RIAS, appeared in the February 1961 issue of the Notices of this Society. Concerning functional analysis it states, in part, ". . . It is these applications to other fields of mathematics that are of greatest interest to Soviet mathematicians. By contrast, American mathematicians are primarily concerned with the structure of functional analysis and the achievement of more abstractness and greater generality. . . . Special mention should be made of the number of excellent textbooks on this subject that have recently been published In the presentation of results and the explanation of current research they are far ahead of anything the west can claim."

This important book by an American mathematician on a branch of functional analysis is a counter-example to the last statement. In the presentation of results and the explanation of current research it measures up to Russian or American or any other standards. As its title indicates, the book is definitely and deliberately written in the "American" tradition. The author's point of view, as stated in the preface, is that "It becomes increasingly evident that, in spite of the deep and continuing influence of analysis on the theory of Banach algebras, the essence of the subject as an independent discipline is to be found in its algebraic development." Accordingly he provides a systematic account of the general theory of Banach algebras emphasizing structure and representation theory. Examples and applications are by no means slighted; indeed a very interesting and valuable

appendix of over fifty pages is devoted to them. Many forward references to this material in the main body of the text aid in providing motivation and illustrations. We proceed to a discussion of the subject matter.

Chapter I. Fundamentals. Notions basic to any discussion of Banach algebras are treated here (e.g. regular and quasi-regular elements, topological divisors of zero and, above all, the spectrum). Elementary proofs, which avoid function theory, are given for the existence of the spectrum, the spectral radius formula and other properties of the spectrum and therefore also for the basic Mazur-Gelfand theorem.

Chapter II. The radical, semi-simplicity and the structure spaces. The algebraic notions of radical, semi-simplicity, primitive rings and ideals and PMI rings are developed along with a discussion of the significance of these ideas in the case of a Banach algebra. This theory is applied to the intriguing uniqueness of norm problem. The spaces of primitive ideals and of maximal modular 2-sided ideals in the hull-kernel topology are introduced; these spaces are later studied in various special cases. A class of algebras which in the commutative case reduce to the regular algebras of Šilov is next taken up; the results here are largely due to Willcox and generalize results obtained by Šilov in the abelian case (further treatment of this subject occurs in the next chapter). The chapter closes with an especially elegant exposition of annihilator algebras.

Chapter III. Commutative Banach algebras. Naturally the Gelfand theory is first presented. A study of algebras of continuous functions on a locally compact Hausdorff space is given featuring the Šilov boundary and other boundaries including the minimal boundary of Bishop. A fairly thorough introduction to the connections with the theory of functions of several complex variables, now being so vigorously pursued, is provided. The important theorem of Šilov that to each compact open subset F of the carrier space there corresponds a non-zero idempotent which assumes the value one precisely on F climaxes this discussion.

Chapter IV. Algebras with involution. This is the longest chapter of the book (99 pages). A theory of $*$ -representations of such Banach algebras as operators on a Hilbert space and their relations to positive functionals is developed. The exposition skillfully incorporates results obtained since the original work of Gelfand and Naimark on this subject. The notion of symmetry is examined in detail. Next the principal properties of B^* -algebras are obtained. Included are proofs that $B^* = C^*$ and of Kadison's remarkable result concerning the equivalence, for $*$ -representations on Hilbert space, of topological and

strict irreducibility. Special results for $*$ -algebras with minimal ideals are obtained; this discussion ends with a development of the basic properties of H^* -algebras.

Appendix. Examples and applications. Most of the details are omitted (with ample reference to the literature). This allows the author to pack considerable meat into these 56 pages. The examples are chosen from algebras of operators and algebras of functions; almost all of them are non-trivial and interesting in their own right. An extensive outline of the theory of group algebras, with emphasis on the non-abelian case, follows. Discussed more briefly are convolution algebras of measures and almost periodic functions on groups.

A lengthy (49 pages) bibliography follows. Throughout the text, notes at the ends of the sections provide the appropriate references to the literature.

As prerequisites for this book, the author assumes a knowledge of the basic facts from the theory of Banach and Hilbert spaces and of the rudiments of modern algebra. By careful organization and exposition he successfully leads the reader to the frontier-of knowledge in the topics of Chapters II, III and IV, in less than 300 pages. In view of the amount of material involved and the close integration necessary, this is a remarkable achievement. It is a well written book and makes pleasant reading. Comparison with original sources shows that the proofs and organization have been thoroughly re-worked and amplified.

As may be expected there are some minor flaws. On page 260 the reference R. B. Smith [1] occurs. But there is no such entry in the bibliography even though R. B. Smith is credited in the preface for his assistance with the bibliography! The reviewer was disconcerted to find the reference to Yood [13] on page 248 in view of the fact that the bibliography lists but 10 items for him. The appropriate reference here is to the *Pacific Journal of Mathematics* vol. 10 (1960) pp. 345–362. This phenomenon occurs only in a few other spots; presumably the text was in a state of constant revision up to the final deadline for the printers.

Naturally there is some overlap with Naimark's *Normed rings*, but each book contains more material outside than inside this intersection. Serious students of the subject should read both.

BERTRAM YOOD

Vorlesungen über Differential- und Integralrechnung. Vol. 1, *Funktionen einer Variablen*. Zweite, neubearbeitete Auflage. By A. Ostrowski. Basel, Birkhäuser, 1960. 330 pp. + 47 fig. 35 s. fr.

The first edition of this book appeared in 1945, and was reviewed