RESEARCH PROBLEMS

1. Richard Bellman: *Differential equations.*

It was shown by Hermite and others that the study of the doubly-periodic solutions of a linear differential equation whose coefficients are analytic doubly-periodic functions of a complex variable is considerably simpler in many ways than the study of the periodic solutions of a linear differential equation with periodic coefficients.

One should in this way be able to obtain excellent approximations to the solution of the Mathieu equation

$$u'' + (a + b \cos 2z)u = 0$$

by considering it as a limiting form of the solution of

$$u'' + (a + b \operatorname{cn} 2z)u = 0$$

as the modulus k^2 tends to zero.

Are there doubly-periodic solutions of the inhomogeneous Van der Pol equation

$$u'' + \lambda(u^2 - 1)u' + u = a \operatorname{cn} \omega z,$$

and can these be used to furnish approximations to the solution of the equation

$$u'' + \lambda(u^2 - 1)u' + u = a \cos \omega z?$$

(Received February 2, 1961.)

2. Richard Bellman: *Asymptotic control theory.*

Consider the problem of determining the minimum of

$$J(u) = \int_0^T (u'^2 + u^2 + u^4)dt$$

over all functions $u(t)$ for which $u(0) = c$. Write $f(c, T) = \min_u J(u)$. It follows from the functional equation approach of dynamic programming that $f(c, T)$ satisfies the nonlinear partial differential equation

$$f_T = \min_v \left[v^2 + c^2 + c^4 + vf_c\right].$$

Since $f(c, T)$ is monotone increasing in T and is uniformly bounded (as we see using the trial function

$$u_0 = \frac{ce^t}{1 + e^{2T}} + \frac{ce^{2T-t}}{1 + e^{2T}},$$

the solution of the corresponding problem where the u^4 term is not present), we expect the limit function $f(c) = \lim_{T\to\infty} f(c, T)$ to satisfy the ordinary differential equation

$$0 = \min_v \left[v^2 + c^2 + c^4 + vf'(c)\right].$$

Establish this and obtain an asymptotic expansion for $f(c, T)$ and for the minimizing function u valid as $T \to \infty$. Generalize by obtaining corresponding results for the minimum of

$$J(u_1, u_2, \cdots, u_N) = \int_0^T [Q(u_1, u_2, \cdots, u_N, u_1', u_2', \cdots, u_N') + P(u_1, u_2, \cdots, u_N)]dt,$$

where Q is a positive definite quadratic form in u_i and u_i' and P is a positive polynomial of higher degree.

Results of this type are important in the modern theory of control processes. (Received February 2, 1961.)