BOOK REVIEWS

Riemann surfaces. By Lars V. Ahlfors and Leo Sario. Princeton University Press, Princeton, New Jersey, 1960. 11+382 pp. $10.00.

For the reader who wants to become acquainted with Riemann surfaces, this book offers an excellent survey of the modern theory. Although it is written as a self-contained treatment for the beginner, it is a pleasure for those already familiar with the subject to read this book, for it is written with the elegance and subtlety that also characterize the *Complex analysis* book by Ahlfors. The book moves at a rather lively pace and one is not encumbered by details that can be supplied by the reader.

The authors' main purpose seems to be the presentation of some of the recent developments in the theory of open Riemann surfaces. As preparation for this, the reader must receive a thorough grounding in topology, and this is just what the first and longest of the five chapters provides. The basic concepts of point set topology are reviewed and surfaces and bordered surfaces are defined. The fundamental group, the index of a curve, the degree of a mapping, and orientability of a surface are introduced and a rather extensive treatment of covering surfaces follows. Simplicial homology theory is presented and the invariance of the 1-dimensional homology group is proved by showing that it is isomorphic to the 1-dimensional singular homology group, which in turn is shown to be isomorphic to the abelianized fundamental group. There follows a very interesting discussion of a compactification of an open surface which distinguishes between the different "components" of the ideal boundary. After classifying polyhedra, a proof of the triangulability of every countable surface is given.

It is not until Chapter 2 that a Riemann surface is defined. The authors present two different proofs of the existence of harmonic and analytic functions on a Riemann surface. In Chapter 2, subharmonic functions are used in the solution of the Dirichlet problem for relatively compact regions with non-empty boundary. This is enough to enable the authors to prove that every Riemann surface is countable, i.e., has a countable base. The general existence theorems for harmonic functions with prescribed behavior is given in Chapter 3 using the method of normal operators of Sario. The canonical mappings of planar (schlichtartig) Riemann surfaces are included along with a discussion of the capacity of boundary components of a Riemann surface.

The main goal of this book is realized in Chapter 4 with the classification theory of open Riemann surfaces. This classification is carried out according to which classes of harmonic or analytic functions the Riemann surface supports. The problem is to find the possible inclusion relations between the different classes of surfaces. The known class inclusions are presented along with counterexamples to show that certain inclusions are not equalities. Many tools, such as the methods of extremal length, harmonic and analytic modules, deep coverings, and triangulations of bounded distortion, are introduced to provide tests for the class of a Riemann surface. There is certainly a wealth of ingenious devices contained in this chapter.

The book closes with a chapter devoted to the more classical theory of closed Riemann surfaces. The existence of harmonic and analytic differentials is repeated, this time using the method of orthogonal projections. A brief synopsis of the theory of abelian integrals on closed Riemann surfaces includes the Riemann-Roch theorem and some of its consequences. Perhaps it should be added that the book contains no lists of problems for the student. It does, however, contain an extensive bibliography.

GEORGE SPRINGER

Anfangswertprobleme bei partiellen Differentialgleichungen. By Robert Sauer. 2d ed. Grundlehren der mathematischen Wissenschaften, vol. 62, Springer Verlag, Berlin-Göttingen-Heidelberg, 1958. DM 38, bound DM 41.

A review of the first edition appeared in volume 59 (1953), of this Bulletin.

Only minor changes have been made in the first four chapters. In the new edition an extensive fifth chapter dealing with the theory and applications of distributions has been added. Distributions are introduced as symbolic derivatives of functions belonging locally to L_1. Their relations to Hadamard's theory of integration of second order equations and to the Riesz method of solution by means of analytic continuation are discussed. Applications of distributions to the solution of some specific problems of supersonic flow are given.

The book is concerned mainly with hyperbolic systems of equations in two independent variables and with problems relating to the wave equation in higher dimensions. Not included are more recent advances in the theory of equations of mixed type and of higher order equations in several variables. Within the limitations of subject matter this slim volume contains a wealth of well integrated material. Analytic constructions of solutions are usually accompanied by dis-