BOOK REVIEWS

The special theory of relativity. By J. Aharoni. New York, Oxford, 1959. 8+285 pp. $7.20.

This book on the special theory of relativity contains, besides the usual subject matter, a chapter on spinors and a chapter on general field theory. The latter discusses the derivation of conservation laws for linear and angular momentum from a Lorentz invariant action principle, and the relation between the canonical and the symmetric energy—momentum tensors.

Any new book on relativity theory invites a comparison with the well-known classics by Weyl, Pauli, and Eddington, and with the more recent books by Bergmann, Landau and Lifshitz, and Synge. The present book does not do well in such a comparison. On the one hand, it is not exhaustive, leaves many loose ends, contains almost no literature references, and thus does not serve the purpose of a reference work. On the other hand, the book is not organized with sufficient care and contains too many misleading statements to serve as a useful text for the serious student who wishes to learn the subject. Here are some examples:

One and a half chapters are devoted to tensors, but there is no proof that contraction is a tensor operation. The process is used constantly and the proof can be given in a few lines. The electromagnetic 4-vector potential is not discussed in the chapter on Maxwell's theory. Instead it is introduced in three lines in connection with variational principles for fields. This is followed by a statement that gauge conditions are in general necessary, when the variational principle does in fact not require such additional conditions. In the chapter on three-dimensional tensors it is suggested "that whenever $A_eB_e=A_{e'}B_{e'}$ and we know that A_e is a vector, we may conclude that B_e is also a vector." Unless A_e is also arbitrary, the conclusion is false. On page 105, an explicit expression for a hyper-surface integral is justified by subdividing the three-dimensional surface into parallelepipeds whose faces are to be respectively parallel to all four coordinate hyperplanes.

ALFRED SCHILD

Algebra. By B. L. van der Waerden. Part 2 (4th ed. of Moderne Algebra) (Grundlehren der mathematischen Wissenschaften, vol. 34) Berlin-Göttingen-Heidelberg, Springer, 1959. 10+275 pp. DM 29.60.

This is the most extensive revision yet made in the classical algebra text. Hints of the fundamental changes begin with the title, which from Moderne Algebra has become Algebra. More hints are found in the updating of terminology—"hypercomplex systems" have become "algebras," "1-isomorphism" is abandoned, "algebraic manifolds" are now usually "varieties" (but free modules are still "Linearformenmoduln"). The significance of the change from "Hilfssatz" in the older material to "Lemma" in the new is lost on this reviewer.

No chapter has been left completely unchanged, but the important changes are the addition of new chapters on algebraic functions of one variable and on topological algebra, the elimination of elimination theory (the remains consist of less than two pages on the existence of a resultant system for homogeneous equations, treated as a consequence of the Nullstellensatz), the complete revision of the chapter on algebras, and some additional material on ideal theory.

The chapter on ideal theory now contains two theorems of Krull: the principal ideal theorem, proved by the use of symbolic powers, and the theorem that the intersection of the powers of an ideal in a local ring must vanish (actually this is done neatly in more generality: $\bigcap \mathfrak{a}^n = \{x \mid (1-a)x=0 \text{ for some } a \in \mathfrak{a}\}$). As might be suspected from this last theorem, the book now defines rings of quotients, even with respect to multiplicatively closed sets containing zero divisors, and uses them more than before, both in fact and in spirit.

The chapter on algebras has one major change in outlook: The Jacobson radical is introduced as the intersection of the annihilators of simple left modules and as the intersection of regular maximal left ideals, which leads quickly to the decomposition of a semisimple ring with minimum condition into a direct sum of simple left ideals without the use of idempotents. The rest of the proof of the Wedderburn-Artin theorems is as before, by noting that the semisimple ring $\mathfrak{o}$ is the endomorphism ring of the completely reducible $\mathfrak{o}$-module $\mathfrak{o}$, and by computation of such endomorphism rings in general. The chapter is prefaced by several sections which may be viewed as edifying examples: Clifford and Grassmann algebras, tensor products, crossed products, and commutative algebras. In this last connection, the theorem that a finite-dimensional commutative algebra over a field is a direct sum of local algebras is proved by writing the algebra as a homomorphic image of a polynomial ring, and using the theory of polynomial ideals; the corresponding theorem for rings with minimum condition does not appear.

The new chapter on algebraic functions of one variable goes as far as the Riemann-Roch theorem, using Weil's proof [Journal für die reine und angewandte Mathematik vol. 179 (1938)]. This necessitates more valuation theory than in volume 1 as well as valuation vectors, and differentials as functions of these vectors. The chapter concludes with a single section on separable generation of function fields of n variables, and a useful section on the connection with the classical Abelian integrals.

The second new chapter defines topological groups, rings, skew fields and vector spaces, beginning from the definition of topological spaces. Next come completions by Cauchy sequences, with a reference to Bourbaki for those who disapprove of countability axioms; a section on finite-dimensional topological vector spaces over topological skew fields (theorems: a linear functional is continuous if and only if its kernel is closed; if a field is complete in a valuation, then a topological vector space over it necessarily carries the Cartesian topology); characterizations of topologies on skew fields which can be given by valuations, in the spirit of Shafarevitch, Kaplansky and Kowalsky-Durbaum; and, finally, Pontryagin's theorem characterizing locally compact skew fields, but in the spirit of the preceding section by proving local boundedness and the existence of an element t with $\lim_n t^n = 0$.

In spite of these additions and revisions, the character of the text remains the same: the same lucid, patient style, the same kind of elegance. The book is essentially self-contained (if one includes volume 1, of course) with the exception of some parts of the chapter on topological algebra where proofs are omitted in favor of references. Every attempt has been made to keep it elementary. However, the meaning of "elementary" is always open to dispute. In the opinion of this reviewer, some topics and treatments which this text still by inference brands as non-elementary are only so because they are so branded. For example, transfinite induction is used, but reluctantly, and Zorn's lemma is only mentioned; hence finiteness or countability hypotheses are imposed in some contexts where they contribute little if anything to ease of presentation or to interest (e.g., only finitely generated semisimple (=completely reducible) modules are studied). In linear algebra there is still much "fixing of ideas" by fixing of bases; e.g., linear, bilinear and quadratic forms are treated primarily as functions of n or $2n$ variables rather than as functions of vectors, at least until after the definition of tensor products in the chapter on algebras (tensor products themselves are defined using bases, though the fundamental universal property appears after a page or two). Thus, in spite of a new section on antisymmetric bilinear forms and the introduction of the Grassmann algebra (in a section called "Examples of noncommutative algebras"), multilinear n-forms are omitted for $n>2$.

The book remains an excellent one for both teacher and student.

The difficulties above can be easily overcome by an alert lecturer, especially with the aid of all the references given to van der Waerden's big, young cousin, Bourbaki.

DANIEL ZELINSKY

Real analysis. By E. J. McShane and T. A. Botts. Princeton, Van Nostrand, 1959. 9+272 pp. $6.60.

As stated in the Preface, "The aim of this book is to present, in a form accessible to the mature senior or beginning graduate student, some widely useful parts of real function theory, of general topology, and of functional analysis." If a mature student, in this context, is understood to be one who has already enjoyed and profited from a substantial introduction to real variables—preferably including some topology and Lebesgue theory—the authors have achieved their objective well. Although material of considerable generality is handled in a style that is frequently quite compact, the proofs and discussion are sufficiently clear and carefully presented to enable the interested reader to follow the argument and to complete any gaps that have been left for him to fill. In the compass of 250 pages the authors lead their audience through the impressive totality of material outlined below.

The book contains eight chapters—numbered 0 through VII—and three appendices. Chapter 0—Preliminaries—sets the stage, with a brief and informal presentation of some of the notation and languages of sets, functions, integers, and the principle of inductive proof. Chapter I—Real Numbers—characterizes the real number system as a complete ordered field (completeness by means of suprema), and introduces partially ordered sets and the maximality principle (a more extended discussion of which is given in Appendix II). Chapter II—Convergence—develops a highly comprehensive limit theory based entirely on the concept of a "direction," that is, a nonempty family of nonempty sets any two of which contain a third, inspired by Moore-Smith generalized convergence. Topological spaces are studied, with uniqueness of limits established for Hausdorff spaces. Compact sets receive special attention. Order-convergence for lattice-valued functions is defined in terms of upper and lower limits (limits superior and inferior). The real number system R and the extended real number system R^* lead to the product spaces R^n and $(R^*)^n$. The Cauchy criterion for convergence of a function from any domain with a direction to a range in R^n is proved. In Chapter III—Continuity—the directions under consideration are specialized either to the family of all relative neighborhoods of a point or (for a nonisolated point) the family of all deleted relative neighborhoods