ences of this type to those given by the author, namely B. H. Neumann's *Essay on free products of groups with amalgamations*, Philos. Trans. Roy. Soc. London no. 919 vol. 246 (1954) pp. 503–554 and M. Lazard's paper *Sur les groupes nilpotents et les anneaux de Lie*, Ann. Sci. Ecole Norm. Sup. (3) vol. 71 (1954) pp. 101–190. These papers may also be considered as monographs on the subjects described in their titles.

The author states that a knowledge of Birkhoff and MacLane's *Survey of modern algebra* will enable a student to read his book. This is true, with respect to both factual knowledge and training in abstract thinking. However, the style of the book is concise and demanding like that of a research paper,— a well written and lucid research paper, but with few "asides" and with little leisurely exposition. (Incidentally, the same may be said about large portions of Burnside's book.) A student who studies Hall's book should find himself well equipped with both the mathematical background and maturity required for the reading of current literature in group theory.

WILHELM MAGNUS

Approximate methods of higher analysis. By L. V. Kantorovich and V. I. Krylov. Translated from the fourth Russian edition by Curtis D. Benster. New York, Interscience Publishers, 1958. xii+681 pp. $17.00.

The publication of an English translation of the remarkable work of Kantorovitch and Krylov is highly to be commended. The first Russian edition appeared in 1936; several subsequent editions, incorporating only minor revisions, appeared from 1941 on. Thus the material is not new. Yet a great part of it has been relatively inaccessible in Russian journals. The remarkably clear and thorough exposition would justify publication of an English version in any case.

The scope of the book is not broad. Its purpose is to study a few approximate methods for solution of the boundary value problems of mathematical physics, to provide in most cases a complete mathematical treatment of the methods, and to expound in full detail the technique of applying them to specific numerical problems. The authors make only brief allusions to digital computers, but it is evident that their procedures have many applications to solution of problems on the modern high-speed machines.

The principal methods studied are the following: series of orthogonal or nonorthogonal functions, infinite systems of simultaneous equations, solution of integral equations by successive approximations and by replacement by finite systems of algebraic equations, finite difference equations, Ritz-Galerkin variational methods, approximate conformal mapping by polynomials, alternating procedure for conformal mapping. The methods are applied mainly to two-dimensional elliptic boundary value problems. In all cases there is a remarkably complete discussion of the error arising from approximation. Many examples are worked out in full numerical detail.

The following is a brief chapter-by-chapter outline: I. INFINITE SERIES: separation of variables for the Dirichlet problem, infinitely many equations in infinitely many unknowns, solution of boundary value problems by nonorthogonal systems of functions, improvement of rate of convergence of series solutions. II. INTEGRAL EQUATIONS: replacement by algebraic equations by means of mechanical quadrature formulas, Neumann series, analytic continuation, reduction to degenerate kernels, method of moments, method of Bateman. III. DIFFERENCE EQUATIONS: interpolation in one and two dimensions, replacement of elliptic boundary value problems in one and two dimensions by difference equations, uniqueness of solution of approximating equations, convergence to solution of original problem. IV. VARIATIONAL METHODS: variational formulation of boundary value and eigenvalue problems, Ritz and Galerkin methods, proof of convergence, applications. V. CONFORMAL MAPPING: approximation by polynomials, extremal properties, perturbation methods, Melentiev's graphical method, Green's function, relation to integral equations, mapping on canonical regions, Schwarz-Christoffel method (in great detail). VI. APPLICATION OF CONFORMAL MAPPING TO BOUNDARY VALUE PROBLEMS. VII. SCHWARZ AND NEUMANN ALTERNATING METHODS: relation to integral equations, proof of convergence.

The translation by C. D. Benster is somewhat literary, but with rare exceptions accurate and clear.

WILFRED KAPLAN

Asymptotic behavior and stability problems in ordinary differential equations. By Lamberto Cesari, Ergebnisse der Mathematik und ihrer Grenzgebiete, No. 16. Berlin-Göttingen-Heidelberg, Springer, 1959, 7+271 pp. DM 68.

To write a Monograph for the Ergebnisse series has common points with writing a seed catalogue—one must tell it all—and writing a treatise one must give a consistent and clear account of the topic. It is particularly arduous when the subject is the one considered here. For in the last two generations or so it has pulled from an almost dormant status to the position of one of the most active chapters of present day mathematics. The pioneers: Poincaré, Liapunov, Birk-