ON OPEN MAPPINGS IN BANACH ALGEBRAS, II

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This paper contains an elementary way to look at the results of [1] and [2]. We prove Theorems 4 and 5 of [2].

Let $\mathfrak{A}$ be a Banach algebra, f a holomorphic function on an open set U in the plane, $a \in \mathfrak{A}$, $\sigma(a) \subset U$, and $f(\sigma(a)) \subset f(V)$ where V is an open set on which f is 1-1. Then a neighborhood of $f(a)$ consists entirely of points of the form $f(b)$ for some $b \in \mathfrak{A}$; further, if $\sigma(a_1) \subset V$ and $f(a) = f(a_1)$, then a and a_1 commute.

To prove this, let $\tilde{f}$ be the restriction of f to V, $\tilde{f}^{\vee}$ the inverse function to $\tilde{f}$. $\tilde{f}^{\vee}$ is defined on an open set W of the plane, with $f(\sigma(a)) \subset W$. There exists a neighborhood of $f(a)$ all of whose elements have spectrum contained in W (*see* e.g., [2, Lemma 2]). For these elements c, set $b = \tilde{f}^{\vee}(c)$. Then $f(b) = \tilde{f}(b) = \tilde{f}(\tilde{f}^{\vee}(c)) = c$.

The second assertion comes from noticing that a commutes with $g(a)$ for any g holomorphic on U; in particular, with $\tilde{f}^{\vee}(f(a)) = \tilde{f}^{\vee}(f(a_1)) = a_1$.

BIBLIOGRAPHY

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