ted is a discussion of systems of n equations, but the case $n=2$ is considered. Another omission is an introduction to the study of the Laplace, wave, and heat equations.

With the exceptions noted above, this book should serve as an excellent introduction to the study of differential equations on the line. The total amount of material covered is just about right for a course of one semester at the senior undergradute or first year graduate level.

Two small slips were noted. On page 64 the author states that $\exp((A+B)x)=(\exp Ax)(\exp Bx)$, where A, B are matrices. This is not in general true, but it is valid in the case $B=-A$ to which it is applied. On page 169, $b(r, t)$ in formula (3.5.2) should be a *vector* of m columns and one row, instead of an m by m matrix as stated.

EARL A. CODDINGTON

Trigonometric series. A survey by R. L. Jeffery. Canadian Mathematical Congress Lecture Series, no. 2, Section III, 1953. University of Toronto Press, Toronto, 1956. 39 pages. $2.50.

Part I is a brief sketch of the high points in the historical development of Fourier series. It is fairly standard except for a discussion of the following problem: Given a function, not necessarily Lebesgue integrable, which is representable as the sum of a trigonometric series, to determine the coefficients. The author indicates the relationship between this and the problem of reconciling the Newton and Leibnitz integrals, in the solutions given by Denjoy, Marcinkiewicz and Zygmund, Burkill, and James. Part II contains detailed proofs of the theorems on Fourier series stated in Part I, as well as a sketch of the methods of James, using his P^2-integral based on the ideas of Perron.

N. J. FINE

Vector spaces and matrices. By Robert M. Thrall and Leonard Tornheim. Wiley. 1957, 12+318 pp. $6.75.

In the preface the authors announce that "In the present textbook we have chosen to proceed simultaneously at two levels, one concrete and one axiomatic. . . . Each new property of a vector space is first discussed at one level and then at the other. . . . We feel that this dual approach has many advantages. It introduces the student to the elegance and power of mathematical reasoning based on a set of axioms and helps to bridge the gap that lies between the pre-eminence of problem solving found in most elementary undergraduate courses and the axiomatic approach that characterizes much modern research in mathematics."

It has long been understood that most courses in matrices have treated the subject in a way that has little relation to the way most working mathematicians think about the topic. More emphasis should be placed on abstract vector spaces and linear transformations and less on the formalisms of the rectangular arrays. It is these ideas rather than the arrays that the student will find so useful in the study of modern mathematics. There exist very good books which provide this emphasis. But all too often students do not take easily to these abstractions and the program outlined by the authors is intended to ease their introduction to such concepts.

While the authors' dual approach is novel in a textbook it is not novel to teaching. Most instructors complement whatever text they are using with their own lectures. Thus it would seem that the authors' approach is sound and that the most serious question concerns the execution of their program. In this they succeed remarkably well, for the book retains a unity and drive toward clearly stated goals despite the seeming burden of parallel discussions.

Actually, the dual program described in the preface is carried out in full detail only in the early chapters. In the later chapters concrete and axiomatic discussions do appear, but there is not the almost self-conscious use of both at each point that characterizes the early chapters. This is really necessary, for otherwise the book would become absolutely unwieldy in size. Furthermore, it has the advantage that the student is led to do more and more thinking for himself as the work progresses.

The first seven chapters cover the material conventionally covered in a semester course in matrices. There is enough material in them that undoubtedly some omissions will be necessary in a one semester course. These seven chapters include: 1. *Vector spaces*, 2. *Linear transformations and matrices*, 3. *Systems of linear equations*, 4. *Determinants*, 5. *Equivalence relations and canonical forms*, 6. *Functions of vectors* (*bilinear and quadratic forms*), 7. *Orthogonal and unitary equivalence*. The pace of the development is accelerated through this part of the book. In the early chapters almost no detail is omitted while in the later chapters many proofs are left to the student. This is very good practice for the student, but it also has the effect of de-emphasizing some important topics, particularly the diagonalization of normal matrices.

The last four chapters are presented on a much more sophisticated level and contain some of the most unusual features of the book. Chapter 8, *Structure of polynomial rings*, contains the decomposition theory for algebras for the case of a polynomial ring over a field modulo an arbitrary fixed polynomial. The beginning student will probably find this chapter tough going, but it is well worth the effort. Chapter 9, *Equivalence of matrices over a ring*, contains the theory of invariant factors. Chapter 10, *Similarity of matrices*, contains the theory of normal forms under similarity transformations in the general case. It is always troublesome to treat this case fully. Here the real effort is invested in Chapter 8 in developing the decomposition theory. This does not make the overall development any easier, if that were the only goal. But it does have the advantage of introducing the student to some of the most important ideas of modern algebra and gives a concrete application of these ideas. Students in applied areas who take courses in matrices are usually not interested in the contents of these chapters in any form. Thus it does seem to be a good idea to cast this material in a form that will have special value for a student whose principal interest is mathematics. Chapter 11, *Linear inequalities*, contains some applications of independent interest, the minimax theorem of game theory and the equivalence of the linear programming problem with the dual linear programming problem. The proofs of these theorems depend less on the previous ten chapters than they do on some elementary topological notions.

The authors place themselves in the camp of those who prefer to write the functional notation with the symbol for the function on the right, i.e., $(x)T$ instead of $T(x)$. Since the two notations are anti-isomorphic, it is purely a matter of preference. But the authors write linear equations in the form $\sum_{\nu=1}^{n} a_{i\nu}x_{\nu}=b_i$ so that in matrix notation a system of equations takes the form $AX=B$. Thus the linear transformation taking X into AX is represented not by A but by its transpose. This awkwardness is avoided by speaking of transformations on spaces of row vectors and column vectors, but the connection between the concrete and the abstract is not as clear here as it should be.

The book is written with great care for accuracy and generality. But in some places the effort to obtain generality with precision results in discussions and notations which are unnecessarily cumbersome. Chapter 3 is that part of the book which suffers most from cumbersome generality. Here discussion of the row-echelon form under proper elementary row operations is alternated with the discussion of the corresponding problem under proper and improper row operations. The two forms differ by some rows of zeros and the confusion of two parallel discussions is not worth the results obtained.

Generally, the exposition is excellent. Objectives are always clearly stated and there is never doubt as to the purpose of any line of development. To assist the beginning student many arguments, particularly mathematical induction arguments, are given in expansive detail. But the authors do this purely for the purpose of instruction and avoid adopting detailed argument as a stylistic burden. Despite taking on this necessary teaching burden the broad outline of the material remains clear. In addition the authors include much discussion of an intuitive nature. There is much to help the student acquire a "feel" for the subject.

In the overall view this book is an exceptionally good one. It is most suitable as a textbook for a better-than-average course that concerns itself with concepts at least as much as with manipulation. The better students will find much in it to stimulate their interest. The book will not appeal to the student impatient with anything that requires understanding as distinct from mere facility. But even the student most narrowly interested only in applications should find the material in the first seven chapters accessible and the viewpoint presented of definite value. The emphasis is on facility through understanding rather than facility through drill. Because the coverage of the material is quite complete with ample explanations, illustrations, and exercises, it is also a good book for the individual student learning on his own. In fact, the book is versatile and suitable for use in a wide array of circumstances and for a wide variety of purposes. It should become a standard in the field.

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