sequential minimax theory for the drift in an additive process—which might have been mentioned.

The material has been carefully planned and presented, and the proofs are neat and compact. Generous use has been made of outside references to most of the more delicate points, and for many of the applications. There is a set of problems at the end of the book, of a wide range of difficulty. There are a number of more or less easily rectifiable misprints.

The following is a reproduction of the table of contents: Chapter 1, Stationary Stochastic Processes and their Representation; Chapter 2, Statistical Questions when the Spectrum is Known; Chapter 3, Statistical Analysis of Parametric Models; Chapter 5, Applications; Chapter 6, Distribution of Spectral Estimates; Chapter 7, Problems in Linear Estimation; Chapter 8, Assorted Problems.

DONALD A. DARLING

Einführung in die Theorie der Differentialgleichungen im reellen Gebiet, by Ludwig Bieberbach, Berlin, Göttingen, Heidelberg, Springer-Verlag, 1956. 8+281 pp. DM 29.80. Bound DM 32.80.

This book, volume 83 in the Grundlehren series, has its genesis in the author's *Theorie der Differentialgleichungen*, which appeared as volume 6 in the same series in 1923. This earlier work had a third edition (1930), and was reprinted by Dover in 1944. Those chapters having to do with differential equations in the complex plane (which included material on analytic equations, regular and irregular singular points) have been expanded into a separate volume, *Theorie der gewöhnlichen Differentialgleichungen*, which appeared as volume 66 of the Grundlehren series. The present volume is an amplification and updating of the remaining chapters of the 1930 work. It is intended as an introduction to the subject of differential equations.

The book is divided into six sections 0–5. The introductory section 0 considers the single equation $dy/dx=f(x, y)$, and by various examples the questions of existence, uniqueness, and the behavior of solutions are posed. The section ends with a proof of the existence and uniqueness theorems assuming f satisfies a Lipschitz condition. Section 1 is an extensive treatment of existence and uniqueness results. It is much more detailed than the corresponding material in the 1930 book (56 pages to 27 pages), and for systems the author introduces vector and matrix notation. The equation $dy/dx=f(x, y)$, where f is continuous, and $|f(x, y)| \leqq M|y| + N$ on $a \leqq x \leqq b$, $|y| < \infty$, is considered. The existence theorem, using the polygonal approximations and the Ascoli lemma, is proved. Uniqueness results of the Osgood and Nagumo variety are given, and a discussion of the maximum and minimum solutions (in cases of non-uniqueness) is given. Other topics treated in this section are continuity in f, initial conditions, and parameters, and linear systems and nth order equations.

The next section 2 on elementary integration methods, and the Runge-Kutta numerical method, is essentially the material in Chapter I, and §7, Chapter II, of the author's 1930 book. The case of a first order system of linear equations with constant coefficients is worked out in detail only for the case of a system of two equations. With very little extra labor the important general case could have been considered.

Section 3 is devoted to a study of autonomous systems close to linear ones. Critical points and the Poincaré-Bendixson theorem are first treated. The behavior of the solutions of the system $dx/dt = ax+by$, $dy/dt = cx+dy$, $ab-bc \neq 0$, is obtained by looking at the six canonical forms. Then the perturbed system $dx/dt = ax+by+p(x, y)$, $dy/dt = cx+dy+q(x, y)$, with $p, q \in C$, $p, q = o(r)$ $(r^2 = x^2+y^2)$ near $(0, 0)$ is treated in great detail. It is shown in each of the six cases just what is required in order to guarantee that the behavior near $(0, 0)$ is dominated by the linear terms. Many examples are given. In the 1930 book it was assumed that p, q were analytic beginning with second degree terms. The Bendixson result on the system $dx/dt = P(x, y)+p(x, y)$, $dy/dt = Q(x, y)+q(x, y)$, with P, Q homogeneous polynomials of degree $m \geqq 1$, $yP-xQ \not\equiv 0$, $p, q = o(r^m)$, is given. New material includes detailed results on the van der Pol, Liénard equatons, and the generalization due to Levinson and Smith, the damped pendulum, as well as some results, for systems of more than two equations, on boundedness and asymptotic stability.

Boundary-value problems for second order equations receive attention in section 4. Much of this appeared in the 1930 book. In particular, the existence of eigenvalues for separated boundary conditions is shown using the Prüfer method, which depends on the oscillation, separation, and comparison theorems. The asymptotic form of the eigenfunctions and eigenvalues is given. In a book of this size it is too bad that a general treatment of regular eigenvalue problems for arbitrary nth order linear equations with self-adjoint boundary conditions could not have been included. An analysis of the periodic solutions of $\ddot{x}+a\dot{x}+bx = p(t)$, a, b constant, p periodic, is given in detail. The geometry of the solutions of the Duffing and Ricatti equations is treated.

Section 5 is a short treatment of partial differential equations of the first order, and this is essentially the same as in the 1930 work. Omitted is a discussion of systems of n equations, but the case $n=2$ is considered. Another omission is an introduction to the study of the Laplace, wave, and heat equations.

With the exceptions noted above, this book should serve as an excellent introduction to the study of differential equations on the line. The total amount of material covered is just about right for a course of one semester at the senior undergradute or first year graduate level.

Two small slips were noted. On page 64 the author states that $\exp((A+B)x)=(\exp Ax)(\exp Bx)$, where A, B are matrices. This is not in general true, but it is valid in the case $B=-A$ to which it is applied. On page 169, $b(r, t)$ in formula (3.5.2) should be a *vector* of m columns and one row, instead of an m by m matrix as stated.

EARL A. CODDINGTON

Trigonometric series. A survey by R. L. Jeffery. Canadian Mathematical Congress Lecture Series, no. 2, Section III, 1953. University of Toronto Press, Toronto, 1956. 39 pages. $2.50.

Part I is a brief sketch of the high points in the historical development of Fourier series. It is fairly standard except for a discussion of the following problem: Given a function, not necessarily Lebesgue integrable, which is representable as the sum of a trigonometric series, to determine the coefficients. The author indicates the relationship between this and the problem of reconciling the Newton and Leibnitz integrals, in the solutions given by Denjoy, Marcinkiewicz and Zygmund, Burkill, and James. Part II contains detailed proofs of the theorems on Fourier series stated in Part I, as well as a sketch of the methods of James, using his P^2-integral based on the ideas of Perron.

N. J. FINE

Vector spaces and matrices. By Robert M. Thrall and Leonard Tornheim. Wiley. 1957, 12+318 pp. $6.75.

In the preface the authors announce that "In the present textbook we have chosen to proceed simultaneously at two levels, one concrete and one axiomatic. . . . Each new property of a vector space is first discussed at one level and then at the other. . . . We feel that this dual approach has many advantages. It introduces the student to the elegance and power of mathematical reasoning based on a set of axioms and helps to bridge the gap that lies between the pre-eminence of problem solving found in most elementary undergraduate courses and the axiomatic approach that characterizes much modern research in mathematics."