cussion in Chapter IV which is concerned with obtaining a representation of a rather complicated kind for a function f in terms of its iterated means over spheres with radii bounded away from zero. By an elementary argument, iterated spherical means are used to obtain solutions of the Euler-Poisson-Darboux equation. Chapter V is dedicated to Asgeirsson's mean-value theorem with applications to the Darboux and wave equations. A generalization to ellipsoidal means due to A. S. Howard is discussed. Chapter VI contains a study of the problem of determining a function from its integrals over spheres of a fixed radius, prefaced by a formal discussion of a more general class of convolution equations.

The differentiability theorems for solutions of various classes of partial differential equations by the use of spherical means which are presented in the last two chapters are the culmination of the techniques expounded in this book. In Chapter VII differentiability theorems are established for solutions of linear and nonlinear elliptic systems by means of the identities of Chapter IV. In Chapter VIII, similar regularity properties are established for the integrals over a family of time-like curves of solutions of nonelliptic partial differential equations. The proofs are carried through in a strikingly explicit and concrete analytical fashion. In the elliptic case, no fundamental solutions are utilized. The results include the differentiability of weak solutions of linear elliptic equations, a fact which plays a crucial role in the L^2-theory of elliptic boundary value problems.

It seems certain that this work by Professor John will occupy an important and permanent position in the literature on partial differential equations.

F. E. BROWDER

Topological transformation groups. By D. Montgomery and L. Zippin. New York, Interscience Publishers, Inc., 1955. 11+282 pp. $5.50.

In writing this book the authors have had two purposes: (1) to present, in connected form, the recent solution by the authors and A. Gleason, based on the work of many mathematicians, of the famous 5th problem of Hilbert, stating that a topological group which is locally homeomorphic to Euclidean n-space E^n is a Lie group; and (2) to report on the work done during the past twenty years on transformation groups, the emphasis being on the way a group acts on a space as a group of transformations.

The first topic represents the final step in a long development to which many outstanding mathematicians have contributed: Introduction of the idea of Lie groups by Lie, via sets of transformations which depend (differentiably) on a number of parameters; intensive study of Lie groups, in particular the classification of all semisimple Lie groups (Killing, Cartan); representation theory of Lie groups (Cartan, Weyl); introduction of the concept of topological group (O. Schreier, 1925); analysis of compact groups (von Neumann, Pontryagin, 1933–1934); analysis of locally compact abelian groups (Pontryagin, 1934); invariant measure in locally compact groups (Haar, 1933); and then about twenty years of study of general locally compact groups (the compactness restriction seemed necessary to get significant results) by many mathematicians; finally, the solution of Hilbert's 5th problem, which actually involves a much more general statement—it might be formulated, somewhat loosely, as saying that in a sense (namely up to certain limit processes) there are no other locally compact groups but our old friends, the Lie groups.—We have sailed across a wide ocean; many times the course was not clear at all; we finally arrived—at our own shores; from a different direction to be sure, but it is the old country. One might almost be a bit disappointed at the outcome; but one should remember that in the years of work on these questions many and very fruitful ideas have been discovered which have enriched all of mathematics tremendously, in addition to the many direct applications of the results.

The first two chapters bring a general introduction to the "classical" theory of topological groups, about what was known in 1935, roughly comparable to the corresponding chapters of the books of Pontryagin and Weil (with some new material added), including things like Haar measure, von Neumann's theorem that a locally compact group which admits a continuous univalent finite dimensional representation is itself a Lie group, the Peter-Weyl theorem, the nature of compact groups, Pontryagin's main theorems on abelian groups. Lie groups are defined but their theory is more or less taken for granted.

Chapters III and IV contain the solution of Hilbert's 5th problem. The chain of reasoning is quite intricate and uses a wide variety of arguments, geometrical and analytical; all the things developed in the introductory section are brought to play. Some of the main steps are: the extension theorem that a group with normal Lie subgroup and Lie factor group is itself a Lie group; the existence of 1-parameter groups in metric connected locally compact groups; the use of these 1-parameter groups for groups without small subgroups (i.e. for groups with a neighborhood of e containing no nontrivial subgroup) to construct a sort of finite dimensional tangent space, in which the group admits a representation [image and kernel are Lie groups, and the extension theorem becomes applicable]; groups without small subgroups are thereby shown to be Lie groups; in the general case, each neighborhood of e contains a subgroup which contains all sufficiently small subgroups of G. The final result can be stated as follows: A locally compact group contains an open subgroup that has arbitrarily small compact normal subgroups with Lie groups as factor groups. Many consequences are derived concerning the topological (local) structure of such groups, etc.

Chapters V and VI are on transformation groups, a subject which, as the authors state, is far from its final phase; in fact the purpose of this part is to make widely scattered material available in convenient form as background for further work. Some knowledge of things like inverse limits, covering spaces, properties of manifolds, fiber bundles etc. is assumed in this part. It is in the nature of the material that there is not so much a systematic development here, but rather a collection of various topics.

Chapter V includes the following (and more): The theorem (Bochner-Montgomery), related to Hilbert's 5th problem, that a locally compact group of C^k-transformations of a C^k-manifold is a Lie group, with the operation being of class C^k in group and space variables simultaneously; Bochner's theorem that a compact group of differentiable transformations with a stationary point can be rendered linear near the stationary point by proper choice of coordinates; Gleason's theorem that there exist local cross sections to the family of orbits under a compact Lie group without fixed points, an important fact for the study of the behavior of the orbits.

Chapter VI is more specialized and concerns mostly the nature of the orbits under a compact group operating on a manifold. Example: If (effective) group and manifold are connected and the orbits are locally connected, then the group is a Lie group (and the orbits are manifolds). The authors' theorem on compact groups acting on Euclidean E^n with at least one $(n-1)$-dimensional orbit is presented: there is a stationary point, and all other orbits are $(n-1)$-spheres. Whether the group is actually linear in suitable coordinates is not quite decided. This is an instance of a question which is the motivation for much of this work: To what extent does the general compact transformation group differ from the "regular" case of a compact group of linear transformations? In contrast to the groups themselves, where, by the solution of Hilbert's 5th problem, irregular behavior is just a matter of change of coordinates, new phenomena do appear for transformation groups (Bing's example). It is still unknown whether an infinite compact totally disconnected group can act effectively on a manifold. The last topic is the authors' theorem that a compact effective group on 3-space E^3 is linear (orthogonal) in suitable coordinates, with an interpretation of this result for the axiomatic foundation of Euclidean geometry of E^3.

The first four chapters are essentially self-contained, up to general mathematical education and some references to special topics. But it is true of course, no surprise with a subject as complex as the one under consideration, that actually a good deal of sophistication and preparation (or perseverance) will be required for appreciation of the material.

Usually things are spelled out in detail, in an almost conversational style. The authors have not aimed at maximum elegance or brevity in their presentation; e.g., some theorems carry unnecessarily strong hypotheses. Occasionally, particularly in the last part, the reader is asked to supply a good deal of the argument.

In both of its main parts the book leads close to present day work; it constitutes a rich source of facts, techniques of all kinds, and references, for anybody who is actively interested in the subject; it will be of great value to beginning and mature mathematicians alike, and is therefore a very welcome addition to the mathematical literature.

HANS SAMELSON

Théorie globale des connexions et des groupes d'holonomie. By A. Lichnerowicz. Edizioni Cremonese, Roma, 1955. 15+282 pp. 4000 lire.

This is a very timely book on the modern theory of connections. The classical theory was mainly initiated by Levi-Civita and Schouten and received, partly because of its applications to the general theory of relativity an extensive development. Elie Cartan observed, from his effective applications of the method of moving frames to various geometrical problems, that the group concept is the basic underlying idea. He knew many examples and had on the basis of this knowledge all the important notions of a general theory, but did not have the tools and terminology to express them. In fact, his "tangent space" is a fiber in the modern terminology, and his space of moving frames is what is now called a principal fiber bundle, etc. This is not to minimize the contributions of modern geometers (Ehresmann, Weil, H. Cartan, Chern, Ambrose, Singer, etc.), whose efforts have made what was once a difficult subject into a beautiful theory. It is now the considered opinion that in differential geometry a connection is a concept pertaining to a principal fiber bundle.

The book is divided into five chapters. Chapters I and II give a