

## RESEARCH PROBLEMS

### 10. R. Bellman: *Probability theory.*

Consider the recurrence relation  $x_{n+1} = ax_n - bx_n^2 + z_n$ ,  $x_0 = c$ , where  $a$  and  $b$  are given parameters and the  $z_k$  constitute a set of random variables drawn from a common distribution. Determine the asymptotic behavior of  $E(x_n)$  under various assumptions concerning  $a$ ,  $b$  and the distribution of the  $z_k$ . (Received February 27, 1956.)

### 11. R. Bellman: *Number theory.*

The relation  $\sum_{k \leq N} 2^{\gamma(k)} \sim N \log N$  (an analogue of a classical device of S. Bernstein in probability theory), where  $\gamma(k)$  represents the number of prime divisors of  $k$ , can be used to show that the number of numbers less than  $N$  which have more than  $c \log \log N$  prime divisors is  $O(N/(\log N)^{c \log 2})$  as  $N \rightarrow \infty$ . Can one substantially improve this estimate for large  $c$ ? (Received February 27, 1956.)

### 12. A. D. Wallace: *A problem on minimax semi-groups.*

Let  $M_n$  be the set of all  $n \times n$  matrices whose entries lie in the closed unit interval and define a multiplication in  $M_n$  by  $(\alpha, \beta)_{ij} = \text{Max}_k (\text{Min} (\alpha_{ik}, \beta_{kj}))$ . Clearly  $M_n$  is a topological semi-group which is homeomorphic with the  $n^2$ -cell. If  $S$  is a topological semi-group which is homeomorphic with an  $n^2$ -cell is it possible to give an abstract description of  $S$  that will identify it with  $M_n$ ? It would be useful to have such a description even at the expense of supplying  $S$  with a dual operation such as obviously exists in  $M_n$ . (Received April 27, 1956.)