BOOK REVIEWS

Begründung der Funktionentheorie auf alten und neuen Wegen. By L. Heffter. Berlin, Springer, 1955. 8+63 pp. 12.60 DM.

The "foundations of function-theory" are understood here to consist of the study of the minimal possible set of assumptions under which a complex function $f(z)$ can be expanded into a power series. Generally such assumptions fall into three broad categories: the existence and continuity of the derivative (Cauchy), the unique existence of the derivative (Goursat), and the unique integrability of $f(z)$ (Morera-Osgood). In the present monograph, the author pictures the foundations of the theory as consisting of six distinct categories with the equivalence of the Cauchy and Weierstrass theories forming one such category, the Cauchy-Goursat theory a second, and the Morera-Osgood theory a third, which is asserted to be " . . . *nur aus historischem Interesse.*" Into a fourth category falls the Looman-Menchoff Theorem and the theory associated with it. The fifth and sixth categories were devised by the author himself in 1936 and 1951, respectively, and each, in its way, can be considered as a variation or extension of the idea of unique integrability of $f(z)$. More specifically, the fifth category consists of showing that if, in a domain G, the real and imaginary parts of $f(z)$ satisfy Cauchy-Riemann difference equations [in terms of mean values of integrals] over every rectangle in G with sides parallel to the axes, then $f(z)$ is regular in G, while the sixth consists of showing that every integral of $f(z)/(z-\zeta)$ vanishes over a certain class of closed paths in G [the point ζ must lie outside the closed path].

Except for the Looman-Menchoff Theorem, the book is completely self-contained, beginning with the notion of the convergence of a sequence and of a Dedekind cut, and proceeding, with all necessary proofs, to Gauss' theorem and the usual properties of functions of a complex variable needed at this stage. All line integrals which are considered are taken either over polygonal paths with segments parallel to one of the axes, or over arcs of circles, a fact which does not compromise generality, but which, in this instance, permits a clearer presentation of the underlying principles. The only theorem which is not proved is the full Looman-Menchoff Theorem which is stated on page 38 together with something of its background; however, a diluted form of the theorem is stated and proved.

The introductory material and the development of the six categories comprise two chapters; in a third (and last) chapter there is a selected list of original papers in chronological order together with the author's comments on these papers. For example, it is of interest to note that the first entry in the list is a letter from Gauss to Bessel, dated 18 December 1811, in which there is communicated the substance of Cauchy's theorem and certain consequences of it. [According to the second entry in the list, this was some three years before Cauchy announced his result and fourteen years before he published it.] When one realizes that the author's first papers in this field appeared almost simultaneously with various papers of Goursat and Morera on the subject, one appreciates the author's connection with the development of the field at the turn of the century.

A. J. LOHWATER

Altes und Neues über konvexer Körper. By H. Hadwiger. (Elemente der Mathematik von höheren Standpunkte aus. Band III.) Basel, Birkhäuser, 1955. 115 pp. 13.50 Swiss fr.

This small volume, containing less than one hundred pages of actual text, gives an elegant and concise account of convex bodies from the standpoint of the geometry of sets. The general approach is to consider the collection of all convex bodies in ordinary space as themselves forming a metric space $\mathfrak{C}$ with convex polyhedra as a dense subset. $\mathfrak{C}$ also has algebraic structure, namely addition (the Minkowski sum of convex sets) and multiplication by positive scalars (dilation). On this space the volume V, surface area A, and integral mean curvature M are functionals defined in the first instance for convex polyhedra and then extended to $\mathfrak{C}$. Thus it is unnecessary to make any assumption beyond convexity itself on the class of bodies considered. In this context many questions concerning, for example, the differential geometry of convex surfaces become unnatural; but, on the other hand, the study of the functionals V, F, M and their properties, Steiner's symmetrization, and so on, are briefly and elegantly treated. Thus the author easily proves a theorem of Gross and Lusternik to the effect that by repeated symmetrization it is possible to gradually transform any convex body into a sphere; he proves Steiner's formula $V(A_\rho)=V(A)+\rho F(A)+\rho^2 M(A)+4\pi\rho^3/3$ for the volume of a convex body A_ρ parallel to A at distance ρ; and he demonstrates the classical Brunn-Minkowski inequalities: $F^2-3MV \geqq 0$ and $M^2-4\pi F\geqq 0$. In this latter connection an interesting discussion is given of the unsolved problem of determining what further inequalities three real numbers V, F, M must satisfy in order to be the values of $V(A)$, $F(A)$, $M(A)$ for some convex body A. The final twenty pages deal in a general fashion with the integral geometry of convex bodies. The book ends with a detailed thirteen page bibliography. Some original results of the author are included, in particular