rather to a series of papers starting in 1948. The footnotes catch up in the long run, if the reader uses his powers of deduction. But more difficult is the deduction that the aforementioned result led to the development of extremal methods and polyhedron packings.

The Minkowskian synthesis (i.e., the packing of space with general convex bodies), certainly receives due attention, although a sense of continuity with earlier work, particularly with that of Hermite, is not made apparent. Likewise the contact between the geometry of numbers and algebraic number theory seems lost in the mass of details again. The reviewer refers to the contact occurring when the algebraist had to work with inequalities not only to find bounds on norms but even to prove the discriminant theorem, which implies the following: "Every irreducible monic polynomial of degree at least two and with integral coefficients contains a repeated (polynomial) factor modulo some prime." Conversely in the last twenty years (owing largely to the influence of Mordell, Davenport, and Mahler), the intense desire for unimprovable bounds in algebraic number theory caused the replacement of the convex body by the more general star body. One contrast in presentation that the reader will immediately note is that while the older methods (Minkowskian and earlier) are often presented with enough detail (diagrams, tables, etc.) to excite the non-specialist, the methods developed in the last twenty years are given in little detail, with emphasis instead on lists of definitions, theorems, and conjectures, and with a valuable bibliography drawn up for 1951. Incidentally, a more modern bibliographical format (alphabetically arranged, and containing the titles of the papers) probably would be more useful.

The chapter titles are as follows: A. Convex bodies. B. Star bodies. C. Linear forms. D. Minima of homogeneous forms. E. Inhomogeneous forms. F. Quadratic definite forms. G. Continued fractions. H. Algebraic numbers. I. Partitions and lattice point figures. The reviewer feels that the inclusion of the last chapter is hardly justified by the historical continuity presented, but that otherwise the titles suggest that the geometry of numbers still would not be beyond Minkowski's recognition.

HARVEY COHN

Economic activity analysis. Ed. by O. Morgenstern. New York, Wiley; London, Chapman and Hall, 1954. 18+554 pp. $6.75.

This is a collection of essays by members of the Princeton University Economics Research Project who were studying the mathematical structure of American type economies with support from the

Office of Naval Research. It is in three parts, prefaced by a valuable summary by the editor. I: Economic properties of the input-output system; II: Mathematical properties of linear economic systems; and III: Metaeconomics.

I. Among the models popular at present is the (static) Leontief system and a dynamic generalization of it that has been studied by the USAF. The basic data for such a system is an input-output matrix (a_{ij}) where the indices correspond to particular industries and a_{ij} represents the effect of an industry i on an industry j. More precisely a_{ij} is the fraction of the output of industry i to industry j. Suppose that the total output of industry i is x_i and suppose that the outside demand for industry i (e.g., for foreign trade) is y_i. Then for equilibrium we have

$$x_i - \sum_j x_j a_{ij} = y_i, \qquad i = 1, 2, \cdots, n,$$

or

$$(I - A)x = y.$$

Thus, given y and knowing $A = (a_{ij})$, we can find $x = (I - A)^{-1}y$.

The coefficients a_{ij} can be obtained from official statistics. The detail in which an economy can be studied depends on the computational power available; early models had $n = 10$, recent models had n about 200, and data are available for n about 500. One of the first problems to be considered is the effect of aggregation, i.e. which industries can be lumped together without serious loss of significance. Effects of various aggregations in reduction of a case of $n = 18$ to a case $n = 8$ are studied in computational detail. Another problem is the effect of uncertainties in the a_{ij}.

II. The a_{ij} just introduced clearly satisfy $a_{ij} \geqq 0$, $\sum_j a_{ij} \leqq 1$. Matrices (a_{ij}) with this property have been called stochastic matrices. They are here called Minkowski-Leontief matrices. The matrix $I - (a_{ij})$ is then called a matrix of Leontief type. It is easily seen that in such a matrix the diagonal element in each row is never smaller than the negative sum of the other elements. Thus matrices of Leontief type are a special case of so-called matrices with dominant main diagonal. These matrices are of great interest in many connections. The Minkowski-Leontief matrices belong to the class of positive or non-negative matrices on which a number of important classical results exist. These theories are here reported and extended in various ways by M. A. Woodbury and Y. K. Wong.

III. The first contribution to this section is a translation of two papers by K. Menger[1] concerned with the validity of the law of diminishing returns. This is: *Let the product of applying* y(e.g., *dollars*) *to* x(e.g., *acres*) *be* $E(x, y)$. *Then for* $h>0$

$$E(x, y_2 + h) - E(x, y_2) < E(x, y_1 + h) - E(x, y_1)$$

provided $y_2>y_1>y=y(x)$. Various properties of production functions E such as monotony, super-additivity, super-homogeneity, dependence are introduced. Examples are presented to show how these are related among themselves and with the above law and similar propositions. These relations are summarized graphically. This essay can be recommended by mathematicians to their friends in social science as an example of how one discipline can be used in another.

The concluding essay by Morgenstern is concerned with the role of experiment and computing in economics. Apart from discussions of a more philosophical nature there are indications of some computational experiments of immediate interest to economists.

OLGA TAUSSKY

Mathematics and plausible reasoning. By G. Pólya. Volume I, *Induction and analogy in mathematics*, 16+280 pp., \$5.50; volume II, *Patterns of plausible inference*, 10+190 pp., \$4.50. Princeton University Press, 1954. 2 vols., \$9.00.

There are many so-called "popular" books on mathematics. Some of them turn out to be of interest to professional mathematicians only (or, perhaps, to professional mathematicians *in ovo* as well). Others are so non-technical as to be well within the reach of any educated layman, and, consequently, their subject hardly deserves to be called mathematics. Most of the time Pólya manages to steer an admirable course between these two extremes. The two volumes under review are, however, not uniform in this respect; the first is more the mathematician's volume and the second the philosopher's. Since this review is addressed to mathematicians, it will discuss the first volume in more detail, and, it may well be charged, with more sympathy, than the second.

The book as a whole is organized around the central thesis that a good guess is quite as important as a good proof. As in his little book *How to solve it*, Pólya advocates that the mathematician should think and talk (at his desk and in the class room) about the theory of guesses as well as the theory of proofs. "Certainly, let us learn proving," he

[1] *Bemerkungen zu den Ertragsgesetzen* and *Weitere Bemerkungen zu den Ertragsgesetzen* in Zeitschrift f. Nationalökonomie vol. 7 (1936) pp. 25–26 and 388–397.