five and seven. There is hardly anything in the book, for instance, on the stability of periodic solutions, or in the sixth chapter on the second order linear equation with periodic coefficients. There is comparatively little reference to work done in the last ten years either in this country or abroad. Aside from these omissions, however, Bellman's book is a pleasant and interesting contribution to the theory of differential equations.

F. HAAS

Discontinuous automatic control. By I. Flügge-Lotz. Princeton University Press, 1953. 8+168 pp. $5.00.

Although self-regulating devices have been in operation since the days of the governor on Watt's steam engine, it is only in recent years that the subject of automatic control has assumed a central position in the engineering and industrial world.

From the mathematical side, the control problem leads to systems of nonlinear differential equations in the following way. If we assume that the state of the physical system is specified at time t by the vector $x(t)$, the study of small displacements from equilibrium gives rise, in a system without control, to a linear vector-matrix equation $\dot{x}=Ax$. If we now consider a system with control, where the control is manifested by a forcing term and the magnitude of the control is dependent upon the state of the system, the resulting equation for x has the form $\dot{x}=Ax+f(x)$, and is, in general, nonlinear.

The term "continuous control" will be used to describe situations in which $f(x)$ is a continuous function of x. In many cases, it was found that continuous control was far too expensive to use. In place then of control devices which gave rise to forcing terms of continuous type, it was far cheaper to design control devices yielding forcing terms whose components are step-functions of x. The simplest version of this type of control system is one with a simple on-and-off control mechanism. This type of control is called "discontinuous automatic control."

A simple example of the mathematical equations which result is the following second order equation, $\ddot{u}+a\dot{u}+bu=c \operatorname{sgn}(\dot{u}+ku)$, where u is now a scalar function. This equation has the form $\ddot{u}+a\dot{u}+bu=c$, over the region of phase space described by $\dot{u}+ku>0$, and the form $\ddot{u}+a\dot{u}+bu=-c$, over the region of phase space described by $\dot{u}+ku<0$. If $\dot{u}+ku=0$, the forcing term is taken to be zero.

We observe then the interesting fact that while the equation itself is nonlinear, over the regions $\dot{u}+ku\gtrless 0$, u may be determined as a solution of a linear equation, albeit a different linear equation over different regions.

This reduces the study of the asymptotic behavior of the solutions of the original nonlinear equation to the study of the iteration of two explicit transformations. These are of simple enough analytic form to permit the use of graphical analysis with great effect. It is this approach which the author has exploited.

Equations of this quasi-linear type are of great interest from the theoretical point of view since they furnish a vital link between the well-regulated world of linearity and the chaotic universe of nonlinearity. It is therefore a valuable contribution to the theory of nonlinear differential equations to have the behavior of the solutions of an important class of these equations presented in as complete and systematic a fashion as is done by the author. References to rigorous proofs of results used in the text, which is aimed at the engineer who must use mathematics, rather than the mathematician who is poaching in the domain of the engineer, are given throughout, particularly to papers of Bilharz, Klotter, Hodapp, and Scholz.

The occurrence of retarded control, which introduces a time-lag in the exertion of the forcing term, gives rise to differential-difference equations in place of the conventional differential equation. There is a brief treatment of this phenomenon in this volume. Those interested in further discussion of the mathematical and engineering consequences of retardation may wish to refer to the papers of Minorsky, cf. Journal of Applied Physics vol. 19 (1948) pp. 332–338, where further references may be found.

The last part of the volume treats the problem of the control of a missile, a problem involving more than one degree of freedom.

The editors of the Princeton University Press are to be congratulated upon adding another attractive and interesting volume to their series on nonlinear mechanics.

RICHARD BELLMAN

Complex variable theory and transform calculus. By N. W. McLachlan. 2d ed. Cambridge University Press, 1953. 11+388 pp. $10.00.

This book is the second edition of a text first published in 1939 (reviewed in Bull. Amer. Math. Soc. vol. 47 (1941) pp. 8–10). The principal changes are in the early, function-theoretic part. The author says that his exposition should now be "rigorous enough for all but the pure mathematician (to whom the book is not addressed)." On the whole this claim seems justified, in some instances more than justified, as on p. 116 where the continuity of a particular entire function is elaborately discussed. There are still mathematical obscurities. For instance, the definition of regular makes $z^{3/2}$ regular at